DOCUMENT RESUME

ED 473 146 SE 067 229

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TITLE	Explorations in Algebra: Hands-On Lab Activities.
INSTITUTION	Hawaii Univ., Honolulu. Curriculum Research and Development Group.
SPONS AGENCY	National Science Foundation, Arlington, VA.
ISBN	ISBN-1-58351-021-4
PUB DATE	2003-00-00
NOTE	189p.
CONTRACT	MDR-8470273
AVAILABLE FROM	Curriculum Research and Development Group, 1776 University Avenue, Honolulu, HI 96822-2463. E-mail: crdg@hawaii.edu; Web site: http://www.hawaii.edu/ crdg.
PUB TYPE	Books (010) -- Guides - Classroom - Teacher (052)
EDRS PRICE	EDRS Price MF01 Plus Postage. PC Not Available from EDRS
DESCRIPTORS	*Algebra; Educational Change; Graphing Calculators; Inquiry; *Learning Strategies; Manipulative Materials; *Mathematics Activities; *Mathematics Instruction; Problem Solving; Secondary Education; Teaching Methods; Worksheets

ABSTRACT

This book is a compatible instructional component to any algebra textbook and was developed by University of Hawaii under the Dwight D. Eisenhower Mathematics and Science Education Improvement Act. The tasks align with the content and instructional approach used in daily classes that emphasize standards-based teaching and learning. The tasks include problem solving, manipulatives, and open-ended questions that let students demonstrate their understanding in different ways. Each topic has multiple labs that can be used at points throughout related chapters giving students the opportunity to enhance their understanding of the concepts or to bridge concepts to skills. Some labs use manipulatives such as algebra tiles or graphing calculators. Each lab includes a problem solving experience. Chapters include: (1) "Problem Solving"; (2) "Real Numbers"; (3) "Algebraic Expressions"; (4) "Equations and Inequalities"; (5) "Graphing"; (6) "Systems of Equations and Inequalities"; (7) "Polynomials"; (8) "Products and Factors"; (9) "Quadratic Equations"; and (10) "Rational Expressions and Equations". (KHR)

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HANDS-ON LAB ACTIVITIES

ED 473 146

EXPLORATIONS IN ALGEBRA

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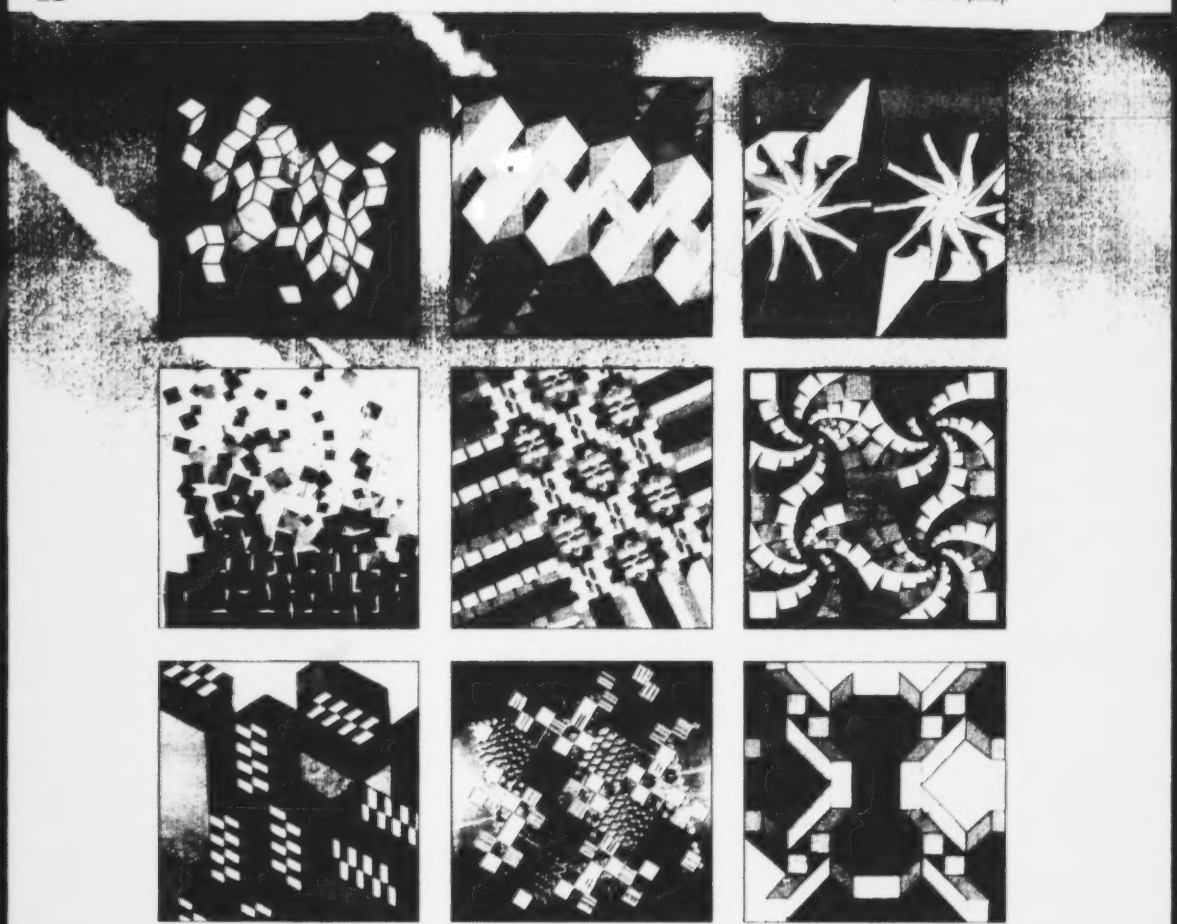

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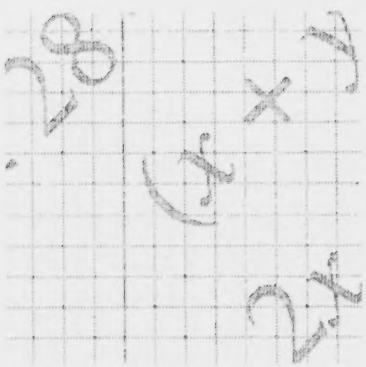
Explorations in Algebra

HANDS-ON LAB ACTIVITIES



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Published by the Curriculum Research & Development Group
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The research underlying this curriculum was supported by the National Services Foundation under Grant MDR-8470273.

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Distributed by
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ISBN 1-58351-021-4

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Introduction

These exploratory labs can accompany any algebra textbook. Funds for developing this package came from the University of Hawai'i and the Dwight D. Eisenhower Mathematics and Science Education Improvement Act.

Overview

Exploratory labs are an instructional component of an Algebra I curriculum. The tasks align with the content and instructional approach used in daily classes that emphasize standards-based teaching and learning. The tasks use problem solving, manipulatives, and open-ended questions that let students demonstrate their understanding in different ways.

Each topic has multiple labs that can be used at points throughout related chapters, giving students the opportunity to enhance their understanding of the concepts or to bridge concepts to skills. Some labs also use manipulatives such as algebra tiles or graphing calculators. Each lab includes a problem-solving experience.

Using the Lab Materials

Each lab presents problems associated with important algebraic concepts and skills. The problems give students the opportunity to work independently as they explore each topic.

Lab tasks reflect the problem-solving processes of reversibility, flexibility, and generalization. The tasks mirror the kinds of problems that promote stronger student understanding.

Selecting Labs. The labs for each topic follow no prescribed order. You can select labs to meet your class's needs. We recommend that you use at least one lab per chapter in your textbook.

If you teach in a setting that uses a block design, such as 2 X 2 or A/B, and algebra is taught over two semesters, you may want to use every lab to provide a mix of activities for your class. In a middle school class, the labs offer variety to keep students' attention as they get hands-on experiences with concrete materials.

Grouping Students. If you use collaborative or cooperative groups as part of your instructional approach, you can use the same groups for the lab activities. The labs suggest using pairs, groups of three or groups of four. Use your discretion in setting up groups for the labs. We do recommend that when groups are suggested, you use some form of a group. This grouping gives students good opportunities to make generalizations or to find good solutions.

Labs should allow time for students to debrief—sharing their findings, giving generalizations, finding similar problems or activities from their daily lessons, and raising questions to consider as they move through the chapter. You should allow about three minutes for students to share in their groups before they share with the class.

While students do the labs, your role is to monitor the groups' activities. The labs are structured so that the directions support groups working independently of you. As groups raise questions, you can field them but avoid giving them too much guidance.

Lab work is collected and scored. We have used several scoring methods. One is to select a random paper from a group and score it, giving each member the same score. Another is to score each paper, awarding grades to individual group members.

Evaluating Labs

Many labs are best scored with a rubric. Rubrics focus on students' thinking and their solution processes rather than solely on their answers. Still, a correct solution or response should be part of the rubric. One useful system is a focused holistic scoring point system.[1] The method focuses on the total solution, evaluating performance with reference to criteria established for scoring from 1 to 5 points. When scoring a paper, you check first to see whether it meets any of the criteria for 5 points. If not, then check the criteria for 4 points, and so on.

5 points Students' lab exploration papers show

- properly labeled responses for all questions.
- appropriate algorithms correctly used.
- correct computations.
- accurate diagrams or drawings.
- conclusions that are supported or justified.

4 points Students' lab exploration papers show

- responses to all questions but with labels missing.
- appropriate algorithms correctly used.
- computations that may be incorrect.
- accurate diagrams or drawings.
- conclusions that are supported or justified.

[1] Charles, Randall, et al. *Problem Solving Experiences in Mathematics, Grades 1-8.* Menlo Park, Calif: Addison-Wesley Publishing Co., 1985.

3 points Students' lab exploration papers show:

- responses given to most questions and labels may be missing.
- algorithms used correctly but inappropriate for the problem, or algorithms appropriate for the problem but used incorrectly.
- conclusions that are not supported.

2 points Students' lab exploration papers show

- answers with no work shown.
- incomplete work with no answer shown.

1 point Students' lab exploration papers show

- no work or answers (a blank paper).
- algorithms used incorrectly and inappropriately for the problem.

This rubric can be applied to any lab you choose to evaluate. Before applying it, make sure you think carefully about each characteristic, changing any that do not fit your class or your approach to instruction.

Before you use the rubric, discuss it with your students so that they understand your expectations. Our students like having a copy of the rubric to refer to as they work. It is important to make the rubric consistent with your daily class expectations. If you focus heavily on student discussion, you will want to make that an integral part of your rubric.

As you use a rubric, keep in mind that a rubric is a set of standards. Each characteristic reveals the quality of students' responses. This view of the evaluation of labs differs from the view that underlies awarding partial credit in that it focuses on quality of the work. Students' work will reflect, in time, the expectations that you set.

LAB 0–A1 NOTES

GETTING TO KNOW YOU

Purpose: Introductory Activity

You can use this lab to set the tone for classroom interactions. It is important for students to feel comfortable with each other if your discussions are to be productive. This lab is best used when students do not know each other. This lab may be interchanged with Lab 0–A2, Find Someone Who . . .

Materials: Getting to Know You (one sheet per student)

Directions: Distribute one sheet to every student. Tell students they have five minutes (or adjust the time to fit your class size) to find students who fit the descriptions on the sheet. Each student can sign for only one item. At the end of the time, the person with the most signatures wins.

Set a timer for five minutes or the time you allotted. At the end of the time, pick some of the attributes and have students share who they found to fit that attribute.

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LAB 0–A2 NOTES

FIND SOMEONE WHO . . .

Purpose: Introductory Activity

You can use this lab to set the tone for classroom interactions. It is important for students to feel comfortable with each other if your discussions are to be productive. This lab is best used when students do not know each other. It may be interchanged with Lab 0–A1, Getting to Know You.

Materials: Find Someone Who . . . (one sheet per student)

Directions: Distribute one sheet to every student. Have students write their responses to the items on the left of the sheet. Allow about five minutes for students to find others in the class who share their same favorites. When they find someone with the same favorite, they should write the first and last name of that student on the right side of the sheet. They should have at least twelve names when they finish the activity.

Set a timer for five minutes or the time you allotted. At the end of the time, pick some of the attributes and have students tell who shared their favorite things.

LAB 0–B NOTES

CALENDAR SCAVENGER HUNT

Purpose: Cooperative Group, Problem Solving

The Calendar Scavenger Hunt is a cooperative group activity. It is best done with groups of four students. If your class is smaller, use groups of three students. This lab gives students an initial problem-solving experience in the early days of the class. Your expectations for students are being developed by the way you introduce and use this lab.

Materials: One set of calendar scavenger hunt clues for each group, calendars for the current year

Note: To prepare the clues, photocopy one set for each group. Cut the clues apart, using the top four clues. The clue about mean average of the numbers is optional, to be used if groups are having difficulty solving the problem. Clip the clues together or put them into an envelope.

Directions: Place students in groups of four. Tell them you are going to give one student in each group a packet of clues. The person getting the packet should take out all the clues and give one to each person in the group. All the clues must be used to solve the problem. Each group member shares its clue by reading it aloud. They may not give away the clue or let someone else read it.

Distribute the packets of clues to the groups. Give the groups about 10 minutes to solve the problem. If one group solves the problem before others, ask them to decide and justify the uniqueness of their answer. That is, is there another solution? Why or why not? If a group is having difficulty, give the fifth clue to the group.

As groups are finishing their work, select two or more groups to write their solutions on transparencies to present on the overhead. Try to select groups that have different solutions OR that used different methods to get the solution.

The solution depends on the year. Some years will have multiple solutions.

LAB 0–A1

Name _____

GETTING TO KNOW YOU

Date _____

Directions: Find people in your class who fit the descriptions below and have them write their names on the line opposite their attributes. Do not use the same person more than once. ***Go!***

You need someone who

1. wears glasses _____
2. plays video games _____
3. has a computer at home _____
4. plays a sport for fun _____
5. plays a musical instrument _____
6. is wearing a ring _____
7. likes vanilla better than chocolate _____
8. is left-handed _____
9. wears contact lenses _____
10. has pierced ears _____
11. is wearing Nike shoes _____
12. is chewing gum _____
13. has two brothers or sisters _____
14. has at least $5.00 _____
15. has traveled out of the country _____
16. takes dance lessons _____
17. has hair parted in the middle _____
18. has a calculator _____
19. receives a weekly allowance of more than $5 _____
20. has at least two keys _____
21. has a birthday the same month as yours _____
22. has a birthday this month _____
23. leaves home before 7:00 a.m. _____
24. speaks a second language _____
25. has eaten sushi _____
26. is wearing a watch _____
27. has a backpack _____
28. is wearing jeans _____
29. reads at least one book a month _____
30. saw a movie last weekend _____

LAB 0–A2

Name _____

FIND SOMEONE WHO . . .

Date _____

Directions: Write down your favorite for each category. Then find *at least* two classmates who have the same favorite you do. Write their first and last names on your sheet. You need to have the names of at least twelve people at the end of the activity.

My favorite FM station is

_____.

Names of classmates

_____

_____

_____

My favorite sport is

_____.

Names of classmates

_____

_____

_____

My favorite flavor of ice cream is

_____.

Names of classmates

_____

_____

_____

My favorite color is

_____.

Names of classmates

_____

_____

_____

My favorite music group is

_____.

Names of classmates

_____

_____

_____

My favorite thing to do on weekends is

_____.

Names of classmates

_____

_____

_____

LAB 0–B

Calendar Scavenger Hunt	Calendar Scavenger Hunt
Jon, Kyle, Reid, and Scott have birthdays in the same month. The sum of their birthdates is 68. When is Jon's birthday?	The birthdates of Jon, Kyle, Reid, and Scott make up the vertices of a rectangle. Scott's birthday is two weeks after Kyle's. When is Scott's birthday?
Calendar Scavenger Hunt	**Calendar Scavenger Hunt**
This year, none of the boys has a birthday on Saturday or Sunday. When is Reid's birthday?	Kyle celebrates his birthday four days after Jon. When is Kyle's birthday?
Calendar Scavenger Hunt	**Calendar Scavenger Hunt**
The mean average of the numbers that lie on the boundary of the rectangle is a date in the center of the rectangle.	**Don't pass this card out!** Depending on the year, this problem may have multiple solutions.

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Chapter 1 Problem-Solving Strategies

Teacher Notes

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LAB 1–A NOTES

BINGO PROBLEM

Purpose:	Problem solving The Bingo Problem is an introductory problem-solving lab that focuses on multiple solution methods. Students use the geometric context to solve the problem.
Materials:	Bingo Problem sheet (one per student), calculator optional
Directions:	Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group. Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. In your group, how many squares can you find on this Bingo card?

   55 squares

2. Describe the method your group used to solve this problem.

   Answers may vary. Students' solution techniques should match their answers in problem 1. Some students may notice that it is the sum of squares matching the dimensions of the larger square. There are twenty-five 1 X 1 squares, sixteen 2 X 2 squares, nine 3 X 3 squares, four 4 X 4 squares, and one 5 X 5 square.

3. How many squares are there on an ordinary checkerboard? (There are eight squares on a side of the board.)

   204 squares

4. How did solving the Bingo problem help you solve the checkerboard problem?

   Answers may vary. Students may use the pattern given in problem 2 or they may report another pattern they used.

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LAB 1–B NOTES

PRODUCT PROBLEM

Purpose:	Problem solving The Product Problem emphasizes using place values for solving nonroutine problems. Students compare their work with other problems that could lead to generalizations.
Materials:	Product Problem sheet (one per student), one calculator per group, number tiles
Directions:	Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group. Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. Using each of the digits 1, 2, 3, 4, 5, and 6 only once, find two 3-digit numbers whose product is as *small* as possible.

   135 and 246

2. What if the digits were 4, 5, 6, 7, 8, and 9? Is there a pattern for finding the smallest product of two 3-digit numbers?

   Answers may vary.

3. What if we were working with addition instead of multiplication? Using the digits 1 to 9 only once, find two 3-digit numbers whose sum uses the remaining three digits.

   Answers may vary. For example,

236	168	391	583	376	637	214
745	327	257	146	542	254	569
981	495	648	729	918	891	783

4. Is your answer unique? If yes, explain why. If no, find other examples.

   As noted in problem 3, the answer is not unique. Students may use different permutations of the digits to form other sums that work.

5. What patterns do you notice?

   Answers may vary. For example,

   - **The sum of the digits of the sum is 18.**
   - **No 1 or 2 can be in the hundreds place of the sum.**
   - **You must use regrouping at least once ("carrying over").**

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LAB 1–C NOTES

DIGITS PROBLEM 1

Purpose: Problem solving

The Digits Problem 1 emphasizes finding patterns. Students have the opportunity to create their own problem after they have experience solving similar ones. This is a reversibility and generalization task.

Materials: Digits Problem 1 sheet (one per student), one calculator per group

Directions: Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. Solve this problem: Two 2-digit numbers have the same digits. The sum of the digits of each number is 10, and the difference between the two numbers is 36. What are the two numbers?

   73 and 37

2. Iwalani tried several two-digit numbers before she found her answer. But when she did the subtraction of each pair, she noticed a pattern. What pattern do you think she noticed? Does the pattern hold for all pairs of two-digit numbers whose sum of the digits is 10?

   Answers may vary. Students may notice that when they subtract the numbers, the difference is a multiple of 9 or a multiple of 18. Or they may notice that the sum of the digits of the difference is 9.

3. Make up a problem similar to the original problem and solve it.

   Answers may vary. Students should solve the problem correctly.

LAB 1–D NOTES

FISH PROBLEM

Purpose: Problem solving

The Fish Problem emphasizes using nonroutine problem-solving strategies. This problem may introduce the notion of uniqueness of answers.

Materials: Fish Problem sheet (one per student)

Directions: Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. How much does a fish weigh if its tail weighs 4 kilograms (kg), its head weighs as much as its tail and half its body, and its body weighs as much as its head and tail together?

   Body is 16 kg, head is 12 kg, and tail is 4 kg, so the fish weighs 32 kg.

2. Describe your solution method.

   Answers may vary. Solution methods should match the solution shown in number 1.

3. Is your answer unique? How do you know?

   Answers may vary. Students should indicate that the answer is unique, but their justifications may differ.

LAB 1–E NOTES

COIN PROBLEM

Purpose: Problem solving

The Coin Problem emphasizes finding patterns as a means for solving a nonroutine problem.

Materials: Coin Problem sheet (one per student), chips or other counters (30 per group)

Directions: Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. Suppose 30 pennies are lined up on a table. Using a supply of nickels, dimes, quarters, and half-dollars, follow these instructions in the order given:
   a. Replace every second coin with a nickel.
   b. Replace every third coin with a dime.
   c. Replace every fourth coin with a quarter.
   d. Replace every fifth coin with a half-dollar.

   How much money is on the table after all the exchanges have been made?

   $5.38

2. What exchanges were made if the total amount on the table after all exchanges is $4.48?

   Start with 30 pennies. Replace every second coin with a half-dollar. Replace every third coin with a nickel. Replace every fourth coin with a dime. Replace every fifth coin with a quarter.

3. What patterns did you notice?

   Answers may vary. Patterns should be consistent with the solutions students found in the previous parts.

LAB 1–F NOTES

FURNITURE PROBLEM

Purpose: Problem solving

The Furniture Problem presents imprecise information. Students are told that the quantities are in a relationship that is not exact. Logic is required to solve the problem, along with number patterns.

Materials: Furniture Problem sheet (one per student)

Directions: Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. The U-Bought-It-U-Own-It Furniture Outlet is having a sale. As a promotion it is advertising some chairs and stools at ridiculously low prices. Although all sale merchandise is sold in "as-is" condition, the outlet promises that each chair will have four legs and each stool will have three legs. The outlet also guarantees that there will be *at least* two more chairs than stools.

   If there are 53 legs all together, how many chairs and stools are being advertised?

   11 chairs and 3 stools

2. What if there were 72 legs and there were *at least* twice as many chairs as there are stools? How many chairs and stools would the outlet have?

   15 chairs and 4 stools; 18 chairs and 0 stools is also a possible answer.

3. Describe how solving problem 1 helped or did not help to solve problem 2.

   Answers may vary. Accept any appropriate description. The description should link to the student's work in problems 1 and 2.

LAB 1–G NOTES

DIGITS PROBLEM 2

Purpose: Problem solving

The Digits Problem 2 extends number patterns. Students experience two extensions of a problem that could lead to generalizations of solution methods.

Materials: Digits Problem 2 sheet (one per student), calculator optional

Directions: Place students in groups of two, three, or four. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. It takes 852 digits to number the pages of a book consecutively. How many pages does the book contain?

   320 pages

2. How many times is the digit 7 printed?

   62 times

3. Is any digit used more times than any other digit? Justify your answer.

   1 is used more often. Students should tell why that is true.

LAB 1–H NOTES

ODD-MAN-OUT

Purpose: Problem solving

The Odd-Man-Out lab uses a game format to introduce a problem that uses working backward as an effective strategy. Students should play the game several times so that they understand the rules before they start the lab.

Materials: Odd-Man-Out sheet (one per student)

Directions: Place students in groups of three. Distribute the problem sheets. Tell students they are to answer each question individually but they can solve the problem within their group. Remind them of the rules of the game because these rules differ from the original game. If students are in groups of four, three students play at a time. Students should take turns playing the game until each person has played at least two rounds.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

Odd-Man-Out is a game for three players in which one person loses and two people win each round. It is a variation of a Japanese children's game called *Jan Ken Po*. At the count of three, each player makes either a closed fist, representing a stone, or shows an open hand, representing paper. If all three players show the same sign, they keep playing until one player shows a sign different from the other two players' signs. The two players with the same sign win the round.

1. Play several rounds of Odd-Man-Out. Record the results for each round. (*Note*: If your group has four people, be sure everyone has a chance to play at least two rounds of the game.)

   Answers will vary as students play the game.

2. Nalani, José, and Vien played three rounds of Odd-Man-Out. To make the game more interesting, they agreed that the loser must pay each winner the amount that the winner already has. At the end of the three rounds, each player has lost one round and has $8. How much money did each player have at the start?

   The first person that lost started with $13, the second loser started with $7, and the third loser started with $4.

LAB 1–I NOTES

STAIRCASE PROBLEM

Purpose: Problem solving

The Staircase Problem gives students an opportunity to problem solve in a geometric context. Have square tiles available for students who need a concrete model of the problem.

Materials: Staircase Problem sheet (one per student), 30 square tiles per group

Directions: Place students in groups of two, three, or four. Distribute the problem sheets and a bag of square tiles per group. Tell students they are to answer each question individually but they can solve the problem within their group.

Allow students 25–30 minutes to complete. Before the period ends, have them share their findings.

1. Sixteen blocks are needed to build a four-step double staircase like the one below. How many blocks will be needed to build a double staircase of similar shape with 50 steps?

   2500 blocks

2. If you had 250 blocks, what would be the maximum number of steps the staircase would have?

   15 steps

3. What is the relationship between the number of blocks needed to build a double staircase and the number of steps in the staircase? Why is this?

   The number of steps squared will give you the number of blocks needed. Students may see that the staircase blocks can be rearranged to form a square whose sides correspond to the number of steps.

4. Write another problem about the double staircase. Solve it.

   Answers may vary. Students should solve their problem correctly.

LAB 1–A

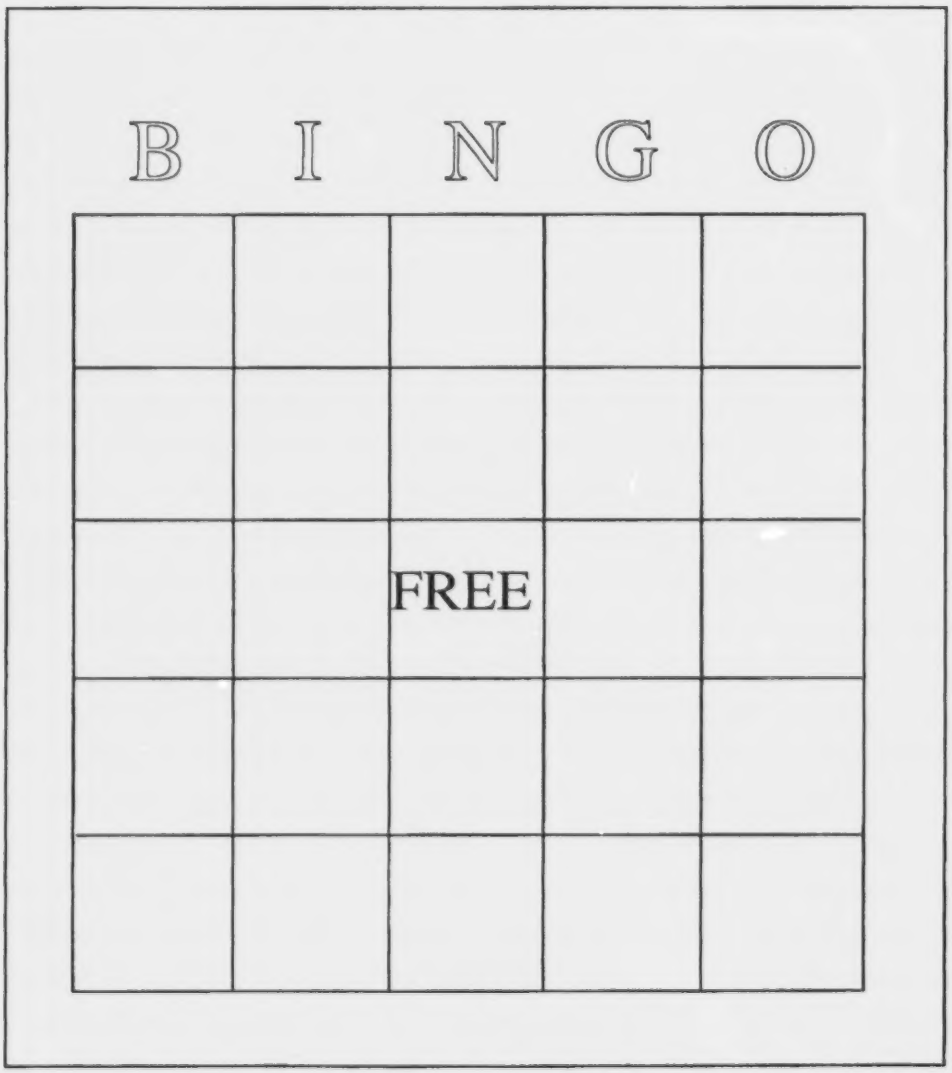
Name _____

Date _____

BINGO PROBLEM

Materials: Calculator optional

1. In your group, how many squares can you find on this Bingo card?



2. Describe the method your group used to solve this problem.

3. How many squares are there on an ordinary checkerboard? (There are eight squares on a side of the board.)

4. How did solving the Bingo problem help you solve the checkerboard problem?

LAB 1–B

Name _____

Date _____

PRODUCT PROBLEM

Materials: Calculator for each group, number tiles

1. Using each of the digits 1, 2, 3, 4, 5, and 6 only once, find two 3-digit numbers whose product is as *small* as possible.

2. What if the digits were 4, 5, 6, 7, 8, and 9? Is there a pattern for finding the smallest product of two 3-digit numbers?

3. What if we were working with addition instead of multiplication? Using the digits 1 to 9 only once, find two 3-digit numbers whose sum uses the remaining three digits.

4. Is your answer unique? If yes, explain why. If no, find other examples.

5. What patterns do you notice?

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LAB 1–C

Name _____

Date _____

DIGITS PROBLEM 1

Materials: Calculator for each group

1. Solve this problem: Two 2-digit numbers have the same digits. The sum of the digits of each number is 10, and the difference between the two numbers is 36. What are the two numbers?

2. Iwalani tried several two-digit numbers before she found her answer. But when she did the subtraction of each pair, she noticed a pattern. What pattern do you think she noticed? Does the pattern hold for all pairs of two-digit numbers whose sum of the digits is 10?

3. Make up a problem similar to the original problem and solve it.

LAB 1–D

Name _____

Date _____

FISH PROBLEM

1. How much does a fish weigh if its tail weighs 4 kilograms (kg), its head weighs as much as its tail and half its body, and its body weighs as much as its head and tail together?

2. Describe your solution method.

3. Is your answer unique? How do you know?

LAB 1–E

Name _____

Date _____

COIN PROBLEM

Materials: Chips or other counters

1. Suppose 30 pennies are lined up on a table. Using a supply of nickels, dimes, quarters, and half-dollars, follow these instructions in the order given:
   a. Replace every second coin with a nickel.
   b. Replace every third coin with a dime.
   c. Replace every fourth coin with a quarter.
   d. Replace every fifth coin with a half-dollar.

   How much money is on the table after all the exchanges have been made?

2. What exchanges were made if the total amount on the table after all exchanges is $4.48?

3. What patterns did you notice?

LAB 1–F

Name _____

Date _____

FURNITURE PROBLEM

1. The U-Bought-It-U-Own-It Furniture Outlet is having a sale. As a promotion it is advertising some chairs and stools at ridiculously low prices. Although all sale merchandise is sold in "as-is" condition, the outlet promises that each chair will have four legs and each stool will have three legs. The outlet also guarantees that there will be *at least* two more chairs than stools.

   If there are 53 legs all together, how many chairs and stools are being advertised?

2. What if there were 72 legs and there were *at least* twice as many chairs as there are stools? How many chairs and stools would the outlet have?

3. Describe how solving problem 1 helped or did not help to solve problem 2.

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LAB 1–G

Name _____

Date _____

DIGITS PROBLEM 2

1. It takes 852 digits to number the pages of a book consecutively. How many pages does the book contain?

2. How many times is the digit 7 printed?

3. Is any digit used more times than any other digit? Justify your answer.

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LAB 1–H

Name _____

Date _____

ODD-MAN-OUT

Directions: Work in groups of three.

Odd-Man-Out is a game for three players in which one person loses and two people win each round. It is a variation of a Japanese children's game called *Jan Ken Po*. At the count of three, each player makes either a closed fist, representing a stone, or shows an open hand, representing paper. If all three players show the same sign, they keep playing until one player shows a sign different from the other two players' signs. The two players with the same sign win the round.

1. Play several rounds of Odd-Man-Out. Record the results for each round. (*Note*: If your group has four people, be sure everyone has a chance to play at least two rounds of the game.)

2. Nalani, José, and Vien played three rounds of Odd-Man-Out. To make the game more interesting, they agreed that the loser must pay each winner the amount that the winner already has. At the end of the three rounds, each player has lost one round and has $8. How much money did each player have at the start?

LAB 1–I

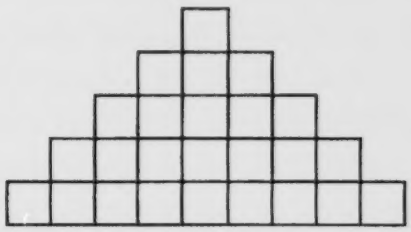
Name _____

Date _____

STAIRCASE PROBLEM

Materials: Color tiles or squares

1. Sixteen blocks are needed to build a four-step double staircase like the one below. How many blocks will be needed to build a double staircase of similar shape with 50 steps?



2. If you had 250 blocks, what would be the maximum number of steps the staircase would have?

3. What is the relationship between the number of blocks needed to build a double staircase and the number of steps in the staircase? Why is this?

4. Write another problem about the double staircase. Solve it.

Chapter 2 The Real Numbers

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Student Worksheet Masters

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LAB 2–A NOTES

PEBBLE MODEL I

Purpose: Using manipulatives

Pebble Model I allows students to form pebble models for a given integer or numerical expression.

Materials: Pebble Model I sheet (one per student), two-color chips or Algebra Tile pieces, notebook paper

Directions: Students work in pairs, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. a. Use your chips to make +5. Draw the model for +5.

Accept any models that accurately represent +5.

b. Draw two more models for +5.

Accept any models that represent +5. For example,
Watch for responses that use only 5 chips separated into two groups.

c. What is the minimum number of chips you need to represent +5? Why?

Students need a minimum of 5 positive chips. Responses should include an explanation that supports why 5 chips are needed.

d. Draw a representation of +5 that uses 7 chips. Now use 9 chips, 12 chips, and 15 chips and draw the representations. What do you notice?

Students should notice that to represent +5, you need to use an odd number of chips, with a minimum of 5 chips. There must be 5 more positive chips than negative chips.

2. a. Use your chips to make -4. Draw the representation for -4.

Answers will vary. Accept any accurate model of -4.

b. Draw two more representations for -4.

Answers will vary. Accept any accurate models of -4 that are different from the models in part 2.a.

c. What is the minimum number of chips needed to represent -4? Why?

Four negative chips are required for the minimum number. Responses should explain why.

LAB 2–A NOTES

2. d. Draw a representation of -4 that uses 6 chips. Now use 8 chips, 12 chips, and 17 chips and draw the representations. What do you notice?

Students should notice that they cannot model -4 with an odd number of chips.

e. Find a representation for -4 that could model this number sentence: $+6 + (-10) = -4$. Explain your representation.

Answers will vary. Representation could include 6 positive chips and 10 negative chips. Students should indicate that 6 zeros are created, leaving four negative chips.

3. a. Show at least three ways to represent 0.

Answers will vary. Any representation should show equal numbers of positive and negative chips.

b. Is 0 positive or negative? Support your answer.

Zero is neither positive nor negative. Responses should explain why.

4. a. How are the representations for $+6$ and -6 alike?

Answers will vary. Students often say that the minimum number of chips needed to represent $+6$ and -6 is six chips. Any other representations require an even number of chips greater than six.

b. How are they different?

Answers will vary. Students often comment that to represent $+6$ requires more positive chips than negative ones. To represent -6 requires more negative chips than positive ones.

5. What ideas have you discovered about positive and negative numbers?

Answers will vary.

LAB 2–B NOTES

PEBBLE MODEL 2

Purpose:	Using manipulatives Pebble Model 2 allows students to model integer operations.
Materials:	Pebble Model 2 sheet (one per student), two-color chips or Algebra Tile pieces, notebook paper
Directions:	Students work in pairs, but each person should answer the problems individually. Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. Kevin used chips to show $-5 + 8 = 3$. Explain how he may have done this.

   Answers will vary. Responses should offer an explanation and possibly a picture representation.

2. a. Shalei let ● represent -1 and ○ represent $+1$. What addition problem could she be showing? Explain.

   Answers may vary. For example, they may say $6 + (-4) = 2$, or $-4 + 6 = 2$.

   b. Write the addition/subtraction fact team that Shalei's chips represent.

   The addition/subtraction fact team is $6 + (-4) = 2$, $-4 + 6 = 2$, $2 - 6 = -4$, and $2 - (-4) = 6$.

   c. Shalei says that her chips can show that $2 - (-4) = 6$. Explain what she means.

   Responses should indicate that to take away four negative chips from two positive ones, zeros must be added until there are four negative chips available to take away.

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LAB 2–B NOTES

3. Show how to use chips to solve these problems:

   a. $-4-(-3)$ b. $-4-3$ c. $-3-4$

   Responses should clearly show an accurate model for each problem. Remind students to show the action used in each operation, such as take away with subtraction. Answers should be clearly identified.

4. Danielle showed $-4-3$ in three steps. Write an explanation for each step.

   Step 1 shows –4. Step 2 shows the addition of zero so that there are enough positive threes to subtract. Step 3 shows three being taken away, or subtracted.

5. What are some important ideas to remember when using chips or pebbles to show addition and subtraction of integers? Provide at least two ideas.

   Answers will vary. Accept responses that are mathematically correct.

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LAB 2–C NOTES

INTEGER PROBLEMS

Purpose:	Problem solving Integer Problems provides opportunities for explaining generalizations about integer operations.
Materials:	Integer Problems sheet (one per student), notebook paper
Directions:	Students work in groups but each person should answer the problems individually. Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. If $\square > 0, \Delta > 0$, what do you know about

   a. $\square + \Delta$?

   The sum should be greater than zero.

   b. $\square \cdot \Delta$?

   The product should be greater than zero.

   c. $\square - \Delta$?

   The difference could be greater than zero if $\square$ is greater than Δ. But if $\square$ is less than Δ, then the difference is less than zero.

2. If $\square < 0, \Delta > 0$, what do you know about

   a. $\square + \Delta$?

   The sum could be greater than zero if the absolute value of Δ is greater than the absolute value of $\square$. But if the absolute value of $\square$ is greater than the absolute value of Δ, then the sum could be less than zero. If the absolute value of Δ is equal to the absolute value of $\square$, then the sum is zero. Students may give cases accompanied by examples.

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LAB 2–C NOTES

2. b. $\Delta - \square$

The difference will always be positive.

c. $\square \bullet \Delta$?

The product is negative.

3. If $\square > 0, \Delta > 0$, what can you say about $\square \div \Delta$?

If $\square$ is greater than Δ, then the quotient will be larger than 1. If $\square$ is less than Δ, then the quotient will be smaller than 1. If the values of the symbols are the same, then the quotient is 1.

4. Tamika said, "$\square$ and Δ are both positive when their difference is positive." Do you agree with Tamika? Why or why not?

This is a reversibility task. Students are given the condition that the difference is positive. However, that does not mean that the numbers you are subtracting will be positive. For example, 1 – (–3) = 4.

LAB 2–D NOTES

WHICH ONE DOESN'T BELONG?

Purpose: Problem solving

Students generalize a rule using real-number contexts.

Materials: Which One Doesn't Belong? sheet (one per student)

Directions: Place students in groups of three or four. Students complete the lab in a group, but they should write their responses individually on their own sheets.

Allow 5–10 minutes at the end of the period to summarize students' findings.

Answers will vary. Students may make other appropriate observations.

1. For each part, circle the selection that does not belong with the other three. Tell why it doesn't belong.

   a. $5 - 8$ $10 - 13$ $15 - 12$ $1 - 4$

   Students may notice that 15 – 12 is the only one that does not have –3 as the difference.

   b. $9 + -4$ $1 + 4$ $0 + 5$ $-1 + 4$

   Students may notice that –1 + 4 is the only one that does not have 5 as a sum.

   c. $4 - 2, 2 - 4$ $5 + 4, 4 + 5$ $3 \times 7, 7 \times 3$ $1 \times 9, 9 \times 1$

   Students may notice that 4 – 2 and 2 – 4 is the only pair that does not have the same answer for both computations. Some students may note that this pair does not show the commutative property.

   d. associative commutative repetitive distributive

   Students may notice that *repetitive* is the only one that does not describe a real-number property.

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LAB 2–D NOTES

1. e. $\frac{1}{2} + \frac{7}{8} = 1\frac{3}{8}$ $\quad 1\frac{3}{8} - \frac{1}{2} = \frac{7}{8}$ $\quad 1\frac{3}{8} - \frac{7}{8} = \frac{1}{2}$ $\quad \frac{7}{8} - 1\frac{3}{8} = \frac{1}{2}$

Students may notice that $\frac{7}{8} - 1\frac{3}{8} = \frac{1}{2}$ is not a true statement. The other three are part of a fact team.

f. $2 \div 0 = 0$ $\quad 2 \times 0 = 0$ $\quad 0 \div 2 = 0$ $\quad 0 \times 2 = 0$

Students may notice that the first equation is not defined and is not part of the fact team represented by the other three equations.

2. Make up a problem like number 1. Describe why one of the parts is different. Describe why the other parts are alike.

Answers will vary. Students' problems should be mathematically correct.

LAB 2–E NOTES

THE PAPER PROBLEM

Purpose: Extending concept development

Students use patterns to solve problems using exponents.

Materials: The Paper Problem sheet (one per student), calculator, paper, ruler, scissors (optional), additional paper (optional)

Directions: Distribute calculators and sheets. Have students work in pairs. Even though they work in pairs, students should individually record their solutions.
The thickness of paper is not given in problem 1. You may have students do this problem first, then give the measurement of 0.003 inches to each group as they finish, or have students devise their own methods of measuring the thickness.

Allow 5–10 minutes at the end of the period for students to summarize their findings.

1. Look at a sheet of notebook paper. If you were to cut the sheet of paper in half, stack one piece on top of the other, cut those in half, stack all the pieces together, repeating this process 20 times, how high, in inches, would you predict the stack of paper would be? Explain how you made your prediction.

   Answers will vary.

2. Find the height of the stack of paper in inches. Show your solution method or describe how you solved it.

   Answers will vary. For example if students use the measurement that a sheet of notebook paper is about 0.003 inches thick, then the stack will be about 3,145.728 inches thick.

3. How does your solution compare to your estimate? Describe.

   Answers will vary. Students may explain that there are 2^{20} sheets of paper.

4. What if you had cut the sheet of paper in fourths, stacked the pieces on top of each other, cut those in fourths, stacked all the pieces together, repeating this process 20 times? How would your results compare? Describe.

   Answers will vary. Students may explain that there are 4^{20} pieces of paper.

LAB 2–E NOTES

5. What patterns do you notice? Describe the patterns in words or with symbols.

Answers will vary.

6. Senika has been offered a job in an expensive restaurant as a waitress. Her boss has offered her two deals: Start with a penny a day and triple her salary each day she works for two months. She will work 20 days each month. Or take the salary of $3000 per month, regardless of the number of days she works. Which offer should she take? Why?

Senika should take the first offer. If she works for only 20 days, she will earn 3^{19} cents or $11,622,614.67.

LAB 2–F NOTES

EXPONENT PROBLEM

Purpose:	Extending concept development Students use patterns to solve problems using exponents.
Materials:	Exponent Problem sheet (one per student), scientific calculator (one calculator per student pair)
Directions:	Distribute calculators and sheets. Have students work in pairs. Even though they work in pairs, students should individually record their solutions. Allow 5–10 minutes at the end of the period for students to summarize their findings.

1. What digit will be in the ones place for
   a. 3^{65}?

      Students may notice that the ones digits of the powers of 3 have a four-number cycle. The cycle is 3, 9, 7, 1, 3, 9, 7, 1, . . . Using this cycle, they may solve the problem by dividing 65 by 4 to find what number in the cycle will represent the ones digit of this power. The answer is 3.

   b. 3^{650}?

      Using the same pattern, students will notice that for the power of 650, it is second in the cycle. The answer is 9.

2. Describe what you noticed about the powers of 3.

   Answers will vary. Students may comment on the cyclic nature of the ones digits in the powers. They may notice other patterns.

3. What digit will be in the ones place for
   a. 8^{65}?

      Students may notice that the ones digits in the powers of 8 are in a cycle. The cycle is 8, 4, 2, 6, 8, 4, 2, 6, . . . To solve the problem, students usually divide by 4 to find that 65 falls on the first number of the cycle, 8.

3. b. 8^{650}?

Using the information about the cycle, students will notice that the 650th power falls on the second digit of the cycle. The answer is 4.

4. How are the powers of 3 and 8 alike and different?

Students may notice that both 3 and 8 have a cycle of four digits. They may add other comments, but encourage them to directly compare the powers.

5. Consider the ones digits of the powers of 4. How are the powers of 4 and the powers of 8 alike and different?

Students may notice that the powers of 4 have a cycle of two digits, 4 and 6, which also belong to the cycle of four digits for the powers of 8.

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LAB 2–G NOTES

FOUR 3s PROBLEM

Purpose: Problem solving

Students use reasoning to create numbers from four 3s and real-number operations.

Materials: Four 3s Problem sheet (one per student), chart paper, colored markers

Directions: Place students in groups of two, three, or four. Give each group one sheet of chart paper and one marker, using different-colored markers to identify the groups. Have students write the numbers 1 to 10 down the left side of the chart paper and record their answers on it.

When groups have completed the lab, have them post their chart paper by taping it on the walls. Groups can compare their representations for numbers. How many unique representations did they find for each number? Were there any numbers they could not find a representation for? Discuss any patterns that students found.

To extend the lab, you may want to leave the chart paper up, adding numbers past 10. Students can fill in their representations as they identify them.

1. On chart paper write other expressions using 4 threes that simplify to each number from 1 to 10. Find at least two expressions for each number.

Answers will vary. These are some possibilities:

1. $3^3 \div 3^3$, $(3+3) \div (3+3)$
2. $(3 \div 3) + (3 \div 3)$, $3 - 3^{3-3}$
3. $3^3 \div (3 \times 3)$, $3 \times 3 - 3 - 3$
4. $3^{3-3} + 3$
5. $(3 + 3) \div 3 + 3$, $3 - (3 \div 3) + 3$
6. $3 \times 3 \div 3 + 3$, $3^3 \div 3 - 3$
7. $3 + (3 \div 3) + 3$, $3 + 3 + (3 \div 3)$
8. $3 \times 3 - (3 \div 3)$
9. $3 \times 3 + (3 - 3)$, $3^{3+3} \times 3$
10. $3 \times 3 + (3 \div 3)$

2. For which of your group's expressions can grouping symbols like parentheses be removed without changing the value? Why?

Answers will vary depending on students' expressions. Check the accuracy of their suggestions by having other groups try evaluating the expressions.

LAB 2–H NOTES

INTEGER REVIEW PROBLEMS

Purpose:	Review If you are teaching Algebra I across three or more semesters, you can use this lab to review integer operations.
Materials:	Integer Review Problems sheet (one per student), chips for pebble model
Directions:	Place students in pairs or small groups. Distribute lab sheets. Ask students to complete the lab. Allow 5–10 minutes at the end of the period to summarize students' solutions.

1. Use the pebble model to show

   a. $-3-(-1)$

   b. $8-6$

   c. $4+7$

   d. $-5+3$

   e. $-2-(-4)$

   f. $8-(-2)$

Answers will vary. Accept any correct representation of the operation. For full credit, answers should be accurate.

2. Explain why this drawing is correct or incorrect:

   ○ = positive pebble ● = negative pebble

   ○○○ − ●● = ○

Answers will vary. However, students' responses should indicate that this representation does not model subtraction. To correctly show the subtraction problem, students would start with 3 white pebbles, then add two zeros (two white and two black pebbles). Then they could take away the two black pebbles, leaving five white pebbles. Positive one is not the correct answer for this problem. Students should see that the model has to be drawn. Mathematical symbols cannot be mixed with a chip representation.

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LAB 2–H NOTES

3. a. Write the other three members of the fact team of $-3 - (-1) = -2$.

The remaining fact team members are $-3 - (-2) = -1$, $-1 + (-2) = -3$, $-2 + (-1) = -3$.

b. Write the other three members of the fact team of $5 \cdot 0 = 0$.

There are only two other members of this fact team. They are $0 \cdot 5 = 0$ and $0 \div 5 = 0$. The "fourth" member, $5 \div 0 = 0$, is not a true statement.

c. Write the other three members of the fact team of $\Delta \cdot \square = \bigcirc$.

The other three members of the fact team are $\square \cdot \Delta = \bigcirc$, $\bigcirc \div \Delta = \square$, and $\bigcirc \div \square = \Delta$. Students should note that neither Δ nor $\square$ can be zero.

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Name _____

LAB 2–A

Group members _____

_____

PEBBLE MODEL 1

_____

Date _____

Materials: Colored chips

Solve as a group. Write your answers on your own paper. Write legibly and explain your answers.

Key

● = –1

○ = +1

1. a. Use your chips to make +5. Draw the model for +5.

   b. Draw two more models for +5.

   c. What is the minimum number of chips you need to represent +5? Why?

   d. Draw a representation of +5 that uses 7 chips. Now use 9 chips, 12 chips, and 15 chips and draw the representations. What do you notice?

2. a. Use your chips to make -4. Draw the representation for -4.

   b. Draw two more representations for -4.

   c. What is the minimum number of chips needed to represent -4? Why?

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d. Draw a representation of -4 that uses 6 chips. Now use 8 chips, 12 chips, and 17 chips and draw the representations. What do you notice?

e. Find a representation for -4 that could model this number sentence: $+6 + (-10) = -4$. Explain your representation.

3. a. Show at least three ways to represent 0.

b. Is 0 positive or negative? Support your answer.

4. a. How are the representations for +6 and –6 alike?

b. How are they different?

5. What ideas have you discovered about positive and negative numbers?

LAB 2–B

PEBBLE MODEL 2

Name _____

Partner _____

Date _____

Materials: Colored chips

Work with a partner. Write your answers on your own paper. Write legibly and explain your answers.

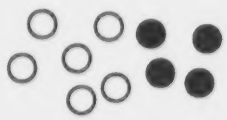
Key

● = –1

○ = +1

1. Kevin used chips to show –5 + 8 = 3. Explain how he may have done this.

2. a. Shalei let ● represent –1 and ○ represent +1. What addition problem could she be showing? Explain.



b. Write the addition/subtraction fact team that Shalei's chips represent.

c. Shalei says that her chips can show that 2 – (–4) = 6. Explain what she means.

3. Show how to use chips to solve these problems:

a. –4 – (–3) b. –4 – 3 c. –3 – 4

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4. Danielle showed $-4 - 3$ in three steps. Write an explanation for each step.

Step 1 Step 2 Step 3

5. What are some important ideas to remember when using chips or pebbles to show addition and subtraction of integers? Provide at least two ideas.

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LAB 2–C

INTEGER PROBLEMS

Name _____

Group members _____

_____

_____

Date _____

1. If $\square > 0, \Delta > 0$, what do you know about

   a. $\square + \Delta$?

   b. $\square \cdot \Delta$?

   c. $\square - \Delta$?

2. If $\square < 0, \Delta > 0$, what do you know about

   a. $\square + \Delta$?

   b. $\Delta - \square$?

   c. $\square \cdot \Delta$?

3. If $\square > 0, \Delta > 0$, what can you say about $\square \div \Delta$?

4. Tamika said, "$\square$ and Δ are both positive when their difference is positive." Do you agree with Tamika? Why or why not?

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LAB 2–D

WHICH ONE DOESN'T BELONG?

Name _____

Group members _____

_____

_____

Date _____

1. For each part, circle the selection that does not belong with the other three. Tell why it doesn't belong.

a. $5 - 8$	$10 - 13$	$15 - 12$	$1 - 4$
b. $9 + -4$	$1 + 4$	$0 + 5$	$-1 + 4$
c. $4 - 2, 2 - 4$	$5 + 4, 4 + 5$	$3 \times 7, 7 \times 3$	$1 \times 9, 9 \times 1$
d. associative	commutative	repetitive	distributive
e. $\frac{1}{2} + \frac{7}{8} = 1\frac{3}{8}$	$1\frac{3}{8} - \frac{1}{2} = \frac{7}{8}$	$1\frac{3}{8} - \frac{7}{8} = \frac{1}{2}$	$\frac{7}{8} - 1\frac{3}{8} = \frac{1}{2}$
f. $2 \div 0 = 0$	$2 \times 0 = 0$	$0 \div 2 = 0$	$0 \times 2 = 0$

2. Make up a problem like number 1. Describe why one of the parts is different. Describe why the other parts are alike.

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LAB 2–E

THE PAPER PROBLEM

Name _____

Group members _____

_____

_____

Date _____

Materials: Calculator, paper, ruler, scissors (optional), additional paper (optional)

1. Look at a sheet of notebook paper. If you were to cut the sheet of paper in half, stack one piece on top of the other, cut those in half, stack all the pieces together, repeating this process 20 times, how high, in inches, would you predict the stack of paper would be? Explain how you made your prediction.

2. Find the height of the stack of paper in inches. Show your solution method or describe how you solved it.

3. How does your solution compare to your estimate? Describe.

4. What if you had cut the sheet of paper in fourths, stacked the pieces on top of each other, cut those in fourths, stacked all the pieces together, repeating this process 20 times? How would your results compare? Describe.

5. What patterns do you notice? Describe the patterns in words or with symbols.

6. Senika has been offered a job in an expensive restaurant as a waitress. Her boss has offered her two deals: Start with a penny a day and triple her salary each day she works for two months. She will work 20 days each month. Or take the salary of $3000 per month, regardless of the number of days she works. Which offer should she take? Why?

LAB 2–F

EXPONENT PROBLEM

Name _____

Group members _____

_____

_____

Date _____

Materials: Scientific calculator

1. What digit will be in the ones place for

   a. 3^{65}?

   b. 3^{650}?

2. Describe what you noticed about the powers of 3.

3. What digit will be in the ones place for

   a. 8^{65}?

   b. 8^{650}?

4. How are the powers of 3 and 8 alike and different?

5. Consider the ones digits of the powers of 4. How are the powers of 4 and the powers of 8 alike? How are they different?

LAB 2–G

FOUR 3s PROBLEM

Name _____

Group members _____

_____

_____

Date _____

Materials: Chart paper, colored markers

By using 4 threes with grouping symbols and the operations +, −, ×, and ÷, we can write an expression that simplifies to the number 1:

$$1 = 3^3 \div 3^3$$

We can also write an expression that simplifies to the number 5:

$$5 = (3 + 3) \div 3 + 3$$

Find other expressions using 4 threes that simplify to each number from 1 to 10.

1. On chart paper write other expressions using 4 threes that simplify to each number from 1 to 10. Find at least two expressions for each number.

2. For which of your group's expressions can grouping symbols like parentheses be removed without changing the value? Why?

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LAB 2–H

INTEGER REVIEW PROBLEMS

Name _____

Group Members _____

_____

_____

Date _____

Materials: Colored chips

1. Use the pebble model to show

Key
● = –1
○ = +1

a. $-3-(-1)$

b. $8-6$

c. $4+7$

d. $-5+3$

e. $-2-(-4)$

f. $8-(-2)$

2. Explain why this drawing is correct or incorrect:

○ = positive pebble ● = negative pebble

○○○ – ●● = ○

3. a. Write the other three members of the fact team of $-3-(-1)=-2$.

b. Write the other three members of the fact team of $5 \cdot 0 = 0$.

c. Write the other three members of the fact team of $\Delta \cdot \square = \bigcirc$.

Chapter 3 The Language of Algebra

Teacher Notes

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Student Worksheet Masters

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LAB 3–A NOTES

PEBBLE MODEL 3

Purpose: Using manipulatives

Pebble Model 3 lets students form tile models for a given algebraic expression.

Materials: Pebble Model 3 sheet (one per student), set of Algebra Tiles or similar pieces
Note: if you use other algebra pieces, you will need to modify the lab.

Directions: Students work in pairs, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. Pick out one each of the following tiles from your set of Algebra Tiles and complete the table below:

	Length in units	Width in units	Area in sq. units
a. small square tile	1	1	1
b. long tile	x	1	x
c. big square tile	x	x	x^2

2. a. Describe how you would use tiles to represent –6.

Answers will vary. For example, 10 red unit tiles and 4 yellow unit tiles.

b. What is the minimum number of tiles you need to model –6? What is the maximum number? What do you notice in your models?

The minimum you need is 6 red unit tiles. There is no maximum number of tiles to model –6. Patterns that students notice will vary.

LAB 3–A NOTES

3. a. Use tiles to model $3x$. Describe or draw your representation.

Answers may vary. The minimum number of tiles to model $3x$ is 3 green long tiles.

b. Can you model $3x$ using 5 tiles? Show the model.

5 tiles:

c. Can you model $3x$ using 7 tiles? 8 tiles? Why or why not? Show your models.

7 tiles:

There must be 3 green long tiles and pairs of "opposite" tiles to be equivalent to $3x$. There will always be an odd number of tiles in the model.

4. a. Show a model for $3x + 2$.

Answers may vary. For example,

b. Show two more models for $3x + 2$. Write algebraic expressions that more accurately represent your two models. Are the two expressions equivalent to $3x + 2$? Why or why not?

Answers may vary. For example,

$-2x + 5x + 2$

5. Use tiles to show a model for twice the sum of $2x + (-1)$. What is the minimum number of tiles needed? Justify your answer.

Answers may vary. The minimum number of tiles needed is 6.

6. What does it mean for two algebraic expressions to be equivalent? Explain by using tiles.

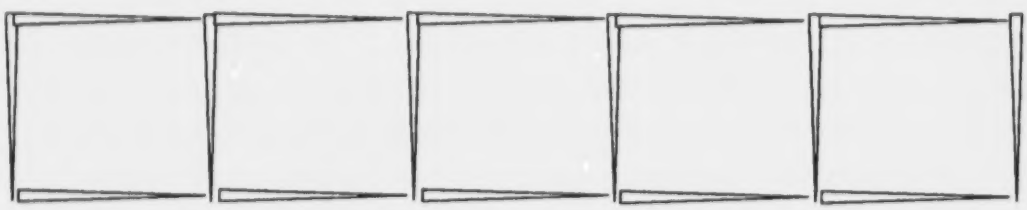
Answers may vary. Students should show that the tile models for equivalent expressions use the same number and type of tiles.

LAB 3–B NOTES

TOOTHPICK PROBLEMS

Purpose:	Extending concept development Toothpick Problems allows students to generalize patterns using algebraic expressions.
Materials:	Toothpick Problems sheet (one per student), package of toothpicks, notebook paper
Directions:	Place students in groups of four and distribute the materials. Students should work as a group, but each person should answer the problems individually. Allow about 10 minutes at the end of the period for students to discuss their solution methods. Encourage students to make generalizations for the counting methods.

1. *Without counting,* find a way to determine how many toothpicks are needed to make this arrangement of five squares:



Answers may vary. For example, students may notice that for 5 squares, there are 5 toothpicks horizontally across the top, 5 toothpicks horizontally across the bottom and 6 toothpicks vertically.

2. Draw a sketch that shows how you got your answer.

Answers may vary. The sketch should reflect the method described in problem 1.

LAB 3–B NOTES

3. Try to find other ways to determine the number of toothpicks needed. Describe your techniques and illustrate them with diagrams.

   a. How are the various methods similar?

   b. How are they different?

   Answers may vary. For example, 4 toothpicks are needed to form the first square, then 3 more toothpicks are needed to form each subsequent square.

4. *Without* actually building it, find a way to figure out how many toothpicks you would need to build a row of 16 squares. Describe the method your group used.

   49 toothpicks are needed. Methods described to determine that number will vary.

5. Ashley has built a row of squares on her desk. She can tell you how many squares there are, but she does not know how many toothpicks she used. Write directions for figuring out how many toothpicks she used.

   Answers will vary. The methods students use should be mathematically appropriate.

6. Make up a new problem by filling in the blanks below. Then solve your problem.
   _____ has _____ toothpicks. What is the largest number of squares in a row that could be built with this number? Justify your answer.

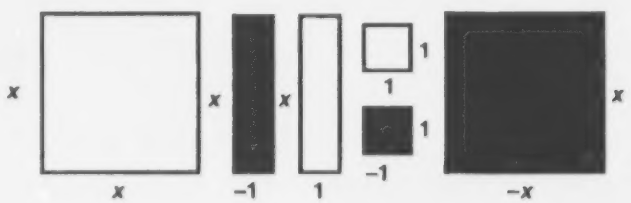
   Be sure a solution, including the method used, is given with the problem.

LAB 3–C NOTES

PEBBLE MODELS FOR EXPRESSSIONS

Purpose:	Using manipulatives Pebble Models for Expressions gives students an opportunity to represent algebraic expressions and explore relationships between them based on their tile models.
Materials:	Pebble Model for Expressions sheet (one per student), set of Algebra Tiles or similar pieces
Directions:	Place students in groups of two, three, or four. Distribute a set of Algebra Tiles to each group and have them complete the activity sheet. Allow 5 to10 minutes at the end of the period for students to discuss their solution methods.

1. The tiles on the right represent 1, –1, x, $-x$, x^2, and $-x^2$. Label the tiles. Explain why you labeled them that way.



Labels and explanations may vary. Examples are shown.

2. a. Use the tiles to show three different representations for $-3x - 2$. Sketch each representation. Write algebraic expressions for each representation.

Answers may vary. For example,

$-4x + x - 2$

b. Are the expressions equivalent? Why or why not?

Yes, all expressions are equivalent. Explanations may vary. For example, since 1 red long tile and 1 green long tile "cancel" each other out, the set of tiles is equivalent to 3 red long tiles and 2 red small square tiles.

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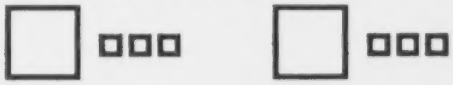
LAB 3–C NOTES

3. a. Write an algebraic expression for "twice the sum of x-squared and 3."

$2(x^2 + 3)$

b. Use the tiles to show a representation of "twice the sum of x-squared and 3." Explain how you decided on what tiles to use.

Answers may vary. For example,



c. What is the minimum number of tiles needed?

The minimum number of tiles needed is 8 (2 blue large square tiles and 6 yellow small square tiles).

d. Write another algebraic expression for the tiles in 3.b.

Answers may vary. For example, $2x^2 + 6$

4. Al said that $6x^2 - 4x$ could be simplified to $2x$. Sal said that $6x^2 - 4x$ was already simplified and used tiles to convince Al. Explain how Sal may have done this.

Answers may vary. For example, Sal may have showed Al that 6 blue large square tiles and 4 red long tiles model the expression $6x^2 - 4x$. Since none of the tiles can be combined or "cancelled", the expression is simplified.

5. Use your tiles to explain how you know when an expression can be simplified. Write your explanation and include your sketches.

Answers may vary. Examples should include tiles that cannot "cancel" or combine with other tiles to form equivalent models.

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LAB 3–D NOTES

GENEROUS FRIEND PROBLEM

Purpose:	Problem solving The Generous Friend Problem enables students to explore multiple solution methods such as working backward or using algebraic expressions and equations.
Materials:	Generous Friend Problem sheet (one per student), calculator (optional)
Directions:	Place students in groups of two, three, or four and give each student a Generous Friend Problem sheet. Students should work as a group, but each person is responsible for answering the problem individually. Encourage multiple solution methods. Allow 5–10 minutes at the end of the period for students to discuss their solution methods.

You walk up to Patrick and say, "Give me as much money as I have now, and I'll give you $10." Patrick gives you the money, and you give him $10.

Next you go to Scharlyne and say, "Give me as much money as I have now, and I'll give you $10." Scharlyne gives you the money, and you give her $10.

Finally you see Valerie and say, "Give me as much money as I have now, and I'll give you $10." Valerie gives you the money and you give her $10.

But now you have no money left. How much money did you start with?

I started with $8.75.

LAB 3–E NOTES

TABLE PROBLEMS

Purpose: Using graphing calculators

Table Problems allows students to look for rules to fit given tables of values, then verify their rules by using the TABLE mode of the graphing calculator.

Materials: Table Problems sheet (one per student), two graphing calculators per group

Directions: Place students in groups of four. Give each group two graphing calculators and each student a Table Problems sheet. Students should work as a group, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. Jeremy set his graphing calculator to the TABLE mode and found that someone had entered these data:

X	Y_1
10	13
11	15
12	17
13	19
14	21
15	23
16	25

   a. Find the rule that may have been used to make this table. Write the rule both in words and with variables.

   Answers may vary. For example, the value of y is seven less than twice the value of x, or $y = 2x - 7$

   b. Describe how your group used the information in the table to find the rule.

   Answers will vary.

LAB 3–E NOTES

2. Kayla set her calculator to ask for the X-value in a table. When it showed $X =$, she entered 11. Y_1 was 10.5. She then entered more values to get the following table:

X	Y_1
11	10.5
22	16
-3	3.5
48	29

a. Kayla noticed that every time she put in an even number, she got a whole number. But when she put in an odd number, she didn't. Kayla thought this pattern would tell her something about the rule. What do you think it would tell her? Support your idea.

Answers will vary. For example, since even numbers are divisible by 2, Kayla might say that the rule requires dividing x by 2.

b. This is Kayla's completed table:

X	Y_1
11	10.5
22	16
-3	3.5
48	29
5	7.5
0	5
6	8

Write a rule that fits Kayla's table.

Answers may vary. For example, the value of y is five more than half the value of x, or $y = 0.5x + 5$

c. Write another rule that fits Kayla's table.

Answers may vary but should be equivalent to those in part b.

LAB 3–E NOTES

3. Fill in the missing information in each table. Write a rule for each table. Then verify the rules using the *Y*= and TABLE features of your graphing calculator.

a.

X	Y_1
0	–2
1	**3**
3	13
5	23
10	**48**
15	**73**
n	**$5n-2$**

b.

X	Y_1
0	3
1	**5**
2	7
3	9
4	**11**
5	13
n	**$2n+3$**

c.

X	Y_1
0	1
1	1.5
2	2
3	**2.5**
4	**3**
5	3.5
10	6
n	**$0.5n+1$**

d.

X	Y_1
0	**0**
1	1
3	**9**
5	25
7	49
15	**225**
–8	**64**
n	**n^2**

4. Describe to a seventh-grader how you find a rule if you're given a table of values.

Answers will vary.

LAB 3–F NOTES

TRAPEZOID TRAINS

Purpose: Using manipulatives

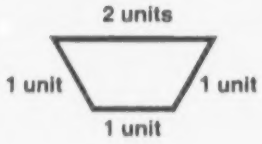
Trapezoid Trains allows students to generalize patterns involving perimeter and area of pattern block figures with algebraic expressions.

Materials: Trapezoid Trains sheet (one per student), 10 red trapezoids, and 10 green triangles from Pattern Blocks

Directions: Place students in groups of four and distribute pattern blocks and Trapezoid Trains sheets. Students should work as a group, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their solutions and methods.

1. Using the length of one side of the triangle as 1 unit, measure the length of each side of the trapezoid. In the space to the right, sketch the trapezoid and label the length of each side.



2. Measure the area of the trapezoid using the triangle as one triangular unit. Record the area of the trapezoid.

3 triangular units

LAB 3–F NOTES

3. Make a trapezoid train. Record the information in the table.

1 2

Number of Trapezoids	1	2	3	4	5	6	10	n
Perimeter of train	5	8	11	14	17	20	32	$3n + 2$
Area of train in triangular units	3	6	9	12	15	18	30	$3n$

4. a. Use this pattern to complete the table:

1 2

Number of Trapezoids	1	2	3	4	5	6	10	n
Perimeter of train	7	11	15	19	23	27	43	$4n + 3$
Area of train in triangular units	5	9	13	17	21	25	41	$4n + 1$

b. Describe the new pattern for perimeter using an algebraic expression. Identify what the variable represents.

$4n + 3$ describes the perimeter of the pattern where n represents the number of trapezoids in the figure.

c. Describe the new pattern for area using an algebraic expression. Identify what the variable represents.

$4n + 1$ describes the area of the pattern where n represents the number of trapezoids in the figure.

LAB 3–G NOTES

POLYHEDRA PATTERNS

Purpose: Using manipulatives

Polyhedra Patterns allows students to build polyhedra models, record data on the number of faces, vertices and edges, and make a generalization that may be Euler's Formula (pronounced "oiler").

Materials: Polyhedra Patterns sheet (one per student), flexible straws, scissors, masking tape or Scotch tape

Directions: Place students in groups of four and distribute the materials. Each student should build at least one of the models shown in the textbook, then work as a group to complete the activity. However, each person should answer the problems individually.

Models can be built using flexible straws. Demonstrate by cutting a slit on the short end of a flexible straw. This makes it easier to insert the short end into the long end of another straw. Slit enough straws to make the desired polygon. Tape completed polygons together to form the desired polyhedron.

Allow 5–10 minutes at the end of the period for students to discuss their models and solutions. Encourage students to express the formula in different forms.

1. Make models of the polyhedra shown using the flexible straws and tape. Label the polyhedra models 1–5.

2. Complete the table below:

Polyhedron	Number of faces	Number of vertices	Number of edges
1	6	8	12
2	4	4	6
3	5	6	9
4	5	5	8
5	8	12	18

LAB 3–G NOTES

3. Describe the relationship you see between the number of faces, vertices, and edges of each polyhedron.

Answers may vary. For example, the number of edges is two less than the sum of the number of faces and the number of vertices of the polyhedron.

4. Create at least three new polyhedra. Label them 6–8. Complete the table below:

Answers will vary.

5. Does the relationship you described in problem 3 apply to your new polyhedra? If not, revise it to work for all eight polyhedra.

The relationship between the number of edges, number of faces, and number of vertices should be the same as described in problem 3.

6. a. Make a polyhedron that has 18 edges and 12 vertices. How many faces, does it have?

Models will vary, but all should have 8 faces.

b. Sketch your polyhedron.

Answers will vary.

c. Compare your polyhedron with those of others in your group. What do you notice?

Answers will vary.

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LAB 3–A

PEBBLE MODEL 3

Name _____

Partner _____

Date _____

Materials: Set of Algebra Tiles or similar pieces

Work with a partner. Record your pair's answers on your own lab sheet.

1. Pick out one each of the following tiles from your set of Algebra Tiles and complete the table below:

	Length in units	Width in units	Area in sq. units
a. Small square tile	1		
b. Long tile	x		
c. Big square tile			

2. a. Describe how you would use tiles to represent –6.

   b. What is the minimum number of tiles you need to model –6? What is the maximum number? What do you notice in your models?

3. a. Use tiles to model $3x$. Describe or draw your representation.

   b. Can you model $3x$ using 5 tiles? Show the model.

3. c. Can you model $3x$ using 7 tiles? 8 tiles? Why or why not? Show your models.

4. a. Show a model for $3x + 2$.

b. Show two more models for $3x + 2$. Write algebraic expressions that more accurately represent your two models. Are the two expressions equivalent to $3x + 2$? Why or why not?

5. Use tiles to show a model for twice the sum of $2x + (-1)$. What is the minimum number of tiles needed? Justify your answer.

6. What does it mean for two algebraic expressions to be equivalent? Explain by using tiles.

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LAB 3–B

TOOTHPICK PROBLEMS

Name _____

Group members _____

_____

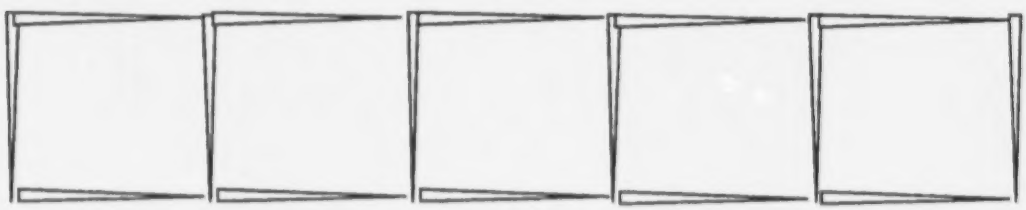
_____

Date _____

Materials: Package of toothpicks, notebook paper

Solve these problems as a group. Write your group's answers on your own sheet of paper.

1. *Without counting,* find a way to determine how many toothpicks are needed to make this arrangement of five squares:



2. Draw a sketch that shows how you got your answer.

3. Try to find other ways to determine the number of toothpicks needed. Describe your techniques and illustrate them with diagrams.

   a. How are the various methods similar?

   b. How are they different?

4. *Without* actually building it, find a way to figure out how many toothpicks you would need to build a row of 16 squares. Describe the method your group used.

5. Ashley has built a row of squares on her desk. She can tell you how many squares there are, but she does not know how many toothpicks she used. Write directions for figuring out how many toothpicks she used.

6. Make up a new problem by filling in the blanks below. Then solve your problem.

   _____ has _____ toothpicks. What is the largest number of squares in a row that could be built with this number? Justify your answer.

LAB 3–C

PEBBLE MODELS FOR EXPRESSIONS

Name _____

Group members _____

_____

_____

Date _____

Materials: Set of Algebra Tiles or similar pieces

Work with a partner. Record your group's answers on your own lab sheet.

1. The tiles on the right represent 1, –1, x, $-x$, x^2, and $-x^2$. Label the tiles. Explain why you labeled them that way.

2. a. Use the tiles to show three different representations for $-3x^2 - 2x$. Sketch each representation. Write algebraic expressions for each representation.

   b. Are the expressions equivalent? Why or why not?

3. a. Write an algebraic expression for "twice the sum of x-squared and 3."

   b. Use the tiles to show a representation of "twice the sum of x-squared and 3." Explain how you decided on what tiles to use.

   c. What is the minimum number of tiles needed?

   d. Write another algebraic expression for the tiles in 3.b.

4. Al said that $6x^2 - 4x$ could be simplified to $2x$. Sal said that $6x^2 - 4x$ was already simplified and used tiles to convince Al. Explain how Sal may have done this.

5. Use your tiles to explain how you know when an expression can be simplified. Write your explanation and include your sketches.

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LAB 3–D

GENEROUS FRIEND PROBLEM

Name _____

Group members _____

_____

_____

Date _____

Materials: Calculator (optional)

Discuss the Generous Friend Problem in your group and write up your group's solution to the problem.

You walk up to Patrick and say, "Give me as much money as I have now, and I'll give you $10." Patrick gives you the money, and you give him $10.

Next you go to Scharlyne and say, "Give me as much money as I have now, and I'll give you $10." Scharlyne gives you the money, and you give her $10.

Finally you see Valerie and say, "Give me as much money as I have now, and I'll give you $10." Valerie gives you the money and you give her $10.

But now you have no money left. How much money did you start with?

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LAB 3–E

TABLE PROBLEMS

Name _____

Group members _____

_____

_____

Date _____

Materials: Two graphing calculators per group

1. Jeremy set his graphing calculator to the TABLE mode and found that someone had entered these data:

X	Y_1
10	13
11	15
12	17
13	19
14	21
15	23
16	25

   a. Find the rule that may have been used to make this table. Write the rule both in words and with variables.

   b. Describe how your group used the information in the table to find the rule.

2. Kayla set her calculator to ask for the X-value in a table. When it showed $X =$, she entered 11. Y_1 was 10.5. She then entered more values to get the following table:

X	Y_1
11	10.5
22	16
-3	3.5
48	29

   a. Kayla noticed that every time she put in an even number, she got a whole number. But when she put in an odd number, she didn't. Kayla thought this pattern would tell her something about the rule. What do you think it would tell her? Support your idea.

b. This is Kayla's completed table:

H	Y_1
11	10.5
22	16
-3	3.5
48	29
5	7.5
0	5
6	8

Write a rule that fits Kayla's table.

c. Write another rule that fits Kayla's table.

3. Fill in the missing information in each table. Write a rule for each table. Then verify the rules using the $Y=$ and TABLE features of your graphing calculator.

a.

X	Y_1
0	–2
1	
3	13
5	23
10	
15	
n	

b.

X	Y_1
0	3
1	
2	7
3	9
4	
5	13
n	

c.

X	Y_1
0	1
1	1.5
2	2
3	
4	
5	3.5
10	6
n	

d.

X	Y_1
0	
1	1
3	
5	25
7	49
15	
–8	
n	

4. Describe to a seventh-grader how you find a rule if you're given a table of values.

LAB 3–F

TRAPEZOID TRAINS

Name _____

Group members _____

_____

_____

Date _____

Materials: 10 red trapezoids and 10 green triangles from Pattern Blocks

Record your group's answers on your own sheet of paper.

1. Using the length of one side of the triangle as 1 unit, measure the length of each side of the trapezoid. In the space to the right, sketch the trapezoid and label the length of each side.

2. Measure the area of the trapezoid using the triangle as one triangular unit. Record the area of the trapezoid.

3. Make a trapezoid train. Record the information in the table.

1 2

Number of Trapezoids	**1**	**2**	**3**	**4**	**5**	**6**	**10**	***n***
Perimeter of train	5		11					
Area of train in triangular units	3	6						

4. a. Use this pattern to complete the table:

1 2

Number of Trapezoids	1	2	3	4	5	6	10	*n*
Perimeter of train								
Area of train in triangular units								

b. Describe the new pattern for perimeter using an algebraic expression. Identify what the variable represents.

c. Describe the new pattern for area using an algebraic expression. Identify what the variable represents.

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LAB 3–G

POLYHEDRA PATTERNS

Name _____

Group members _____

_____

_____

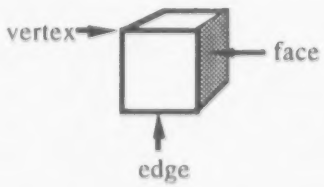
Date _____

Materials: Paper towel, flexible straws, scissors, masking tape or Scotch tape, student textbook

Work with the other students in your group. Write your answer on your own paper.

1. Make models of the polyhedra shown using the flexible straws and tape. Label the polyhedra models 1–5.

2. Complete the table below:



Polyhedron	Number of faces	Number of vertices	Number of edges
1			
2			
3			
4			
5			

3. Describe the relationship you see between the number of faces, vertices, and edges of each polyhedron.

4. Create at least three new polyhedra. Label them 6–8. Complete the table below:

Polyhedron	Number of faces	Number of vertices	Number of edges
6			
7			
8			

5. Does the relationship you described in problem 3 apply to your new polyhedra? If not, revise it to work for all eight polyhedra.

6. a. Make a polyhedron that has 18 edges and 12 vertices. How many faces does it have?

   b. Sketch your polyhedron.

   c. Compare your polyhedron with those of others in your group. What do you notice?

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Chapter 4 Equations and Inequalities in One Variable

Teacher Notes

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Student Worksheet Masters

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LAB 4–A NOTES

EXPRESSIONS AND EQUATIONS

Purpose: Using manipulatives

Expressions and Equations gives students an opportunity to represent algebraic expressions with tiles and to use models to solve equations and inequalities.

Materials: Expressions and Equations sheet (one per student), Algebra Tiles, notebook paper

Directions: Place students in groups of two and distribute the materials. Students work in pairs but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their solutions to problems 2 and 3.

1. a. Use tiles to represent the expressions given. On a separate sheet of paper, draw a sketch of each representation.

Answers may vary. For example,

1) $2x + 3$

2) $2(x + 3)$

3) $3x + 2(x + 3)$

4) $3x - 2(x + 3)$

b. Simplify each expression in 1.a. Support your answer with the tile representations.

1) $2x + 3$
2) $2x + 3$
3) $5x + 6$
4) $x - 6$

LAB 4–A NOTES

2. Doug used tiles to help him solve the equation $2(x + 3) + x - 2 = 2x + 7$.

He said he thought of a balance scale.

Explain how Doug may have used the tiles to solve the equation.

Answers may vary. For example, Doug may notice that both sides of the balance have 2 *x*-tiles and 6 positive unit tiles in common. He can remove these tiles from both sides of the balance scale and maintain the balance. He can then see that 1 *x*-tile and 2 negative unit tiles on the left side must balance with 1 positive unit tile on the right side. Thus the long tile must be equivalent to 3 positive unit tiles, or $x = 3$.

3. How could Doug solve the inequality $2(x + 3) + x - 2 > 2x + 7$ using the balance scale model?

Answers may vary. For example, since the left side of the balance is greater than the right side, there must be more than 3 positive units on the left side. Students may use flexibility from problem 2.

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LAB 4–B NOTES

USING THE TI CALCULATOR

Purpose: Using graphing calculators

Using the TI Calculator introduces students to using the [TABLE] mode of the graphing calculator to solve equations.

Materials: Using the TI Calculator sheet (one per student), TI-80, 82, or 83 calculator, notebook paper

Directions: Place students in groups of two and distribute materials. Students work in pairs, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to summarize what they learned about using the graphing calculator.

1. Erin used a calculator to help her solve the equation $3x - 5x + 4 = 8$. This is what she did:

   1) She turned on the calculator, then hit the [Y=] key.
   2) Then she entered **$3x - 5x + 4$** where the blinking cursor was.
   3) Next she pressed the [2nd] key and then the [GRAPH] key. What happened?
      A two-column table with the headings *x* and y_1 appears.
   4) Erin also experimented with the blue arrow keys. What did these keys do?
      The cursor moved in the direction of the blue arrow key pressed.
   5) Describe how you think Erin was able to solve the equation.
      Answers may vary. Erin can use the blue arrow keys to move the cursor in the y_1-column until the value 8 is highlighted. She can then look at the corresponding value in the *x*-column to find the solution to $3x - 5x + 4 = 8$.

2. How could you use Erin's method to solve $5(x + 4) - 1 = 9$? Explain thoroughly.

   Answers may vary. For example, Erin can hit the [Y=] key and enter $5(x + 4) - 1$. Then she can press the [2nd] key and then the [GRAPH] key to display a table of values. Erin can then use the blue arrow keys to move the cursor in the y_1-column until the value 9 is highlighted. She can then look at the value in the *x*-column to find the solution to the equation $5(x + 4) - 1 = 9$.

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LAB 4–B NOTES

3. Patrick discovered the [QUIT] and the [TBLSET] commands on the calculator. How did he access them and what does each do?

Answers may vary. For example, Patrick can access [QUIT] by pressing the [2nd] key and then the [MODE] key to clear the display and return to the home screen. He can press the [2nd] key and then the [GRAPH] key to change the settings for the table display.

4. Chelsea tried solving $6x - 2(x + 2) = 6$ using Erin's method but was unable to find the solution. Kris suggested using [TBLSET] and then changing what ΔTBL equaled. Explain what she may have done.

Answers may vary. For example, Chelsea can input $\Delta TBL = 0.5$ then press the [2nd] key and then the [GRAPH] key.

5. Ryan made use of Y_1 and Y_2 in the [Y=] screen on the calculator to solve the following problems:

Solve for f:

a. $2f + 3 = 28 - 3f$
b. $2f + 3 < 28 - 3f$

What do you think Ryan did? How was he able to access the Y_2 column in the table?

Answers may vary. For example, Ryan can input $Y_1 = 2x + 3$ and $Y_2 = 28 + 3x$. For part a., he might look at the table, find where Y_1 and Y_2 have the same value, and look at the corresponding value of x to determine $x = 5$. For part b., he might conclude that $x < 5$ since the values of Y_1 are less than the values of Y_2 for those values of x.

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LAB 4–C NOTES

EQUATION PROBLEM

Purpose:	Extending concept development Equation Problem allows students to solve equations in more than one way and to change the subject of an equation.
Materials:	Equation Problem sheet (one per student), Algebra Tiles, graphing calculator,
Directions:	Place students in groups of four and distribute materials. Students will work in groups but each person should answer the problems individually. Allow about 10 minutes at the end of the period for students to discuss their methods of solving the problems.

1. You have several methods of solving equations. You can use a table, guess-and-check, algebra, diagrams, or a combination of these. Select a method and solve: $4x - 5 = 9 + 2x$.

   Methods will vary. Solution to the equation should be correct. The method should be applied correctly to the equation.

   Solve it another way.

2. Simplify: $3(x + 2) - 4(5 - x)$.

   $7x - 14$
   Students must show or describe how they simplified the expression.

   You can use the calculator to check your simplified expression. Pick a value for x, and use your calculator to check. Show your displays below.

   Answers may vary. Students should show a table of values and indicate how they used the table to solve the equation.

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LAB 4–C NOTES

3. You've seen devices called *traffic counters* that count the vehicles passing along a street. You can recognize the devices as thin black cords stretched across a street or highway and connected to a "brain box" at the side of the road. The traffic counter is usually pressure activated. It registers a count each time a set of wheels (wheels on a single axle) rolls over the tube. A normal car registers two counts: one for the front wheels and one for the rear wheels. A truck with three axles (front wheels plus a double set of rear wheels) registers three counts. A large semi might register four or five counts.

   Suppose that during a one-hour period, a particular traffic counter registers 35 counts on a residential street where heavy trucks are prohibited. (That is, only two-axle vehicles like cars or three-axle vehicles like light trucks are allowed.) How many cars and light trucks passed over the traffic counter? (Show your work, and all solutions.)

   Methods may vary. The possible solutions are displayed in the table below, where c is the number of cars and t is the number of trucks.

c	16	13	10	7	4	1
t	1	3	5	7	9	11

4. Sometimes an equation can be used to show the relationship in the problem. In this problem, the equation would be $2c + 3t = 35$. Using any operations solve this equation for c. That is, your final equation should have c on the left and everything else on the right of the equal sign. This is called making c the ***subject*** of the equation.

   $c = (35 - 3t) \div 2$

   Describe what you did to write the equation in the form $c = ?$.

   Methods may vary.

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LAB 4–D NOTES

INEQUALITY LAB

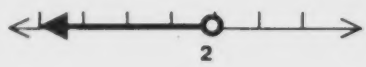
Purpose:	Extending concept development The Inequality Lab allows students to compare number line graphs of inequalities with a table of values that make them true or false.
Materials:	Inequality Lab sheet (one per student)
Directions:	Students work in pairs, but each person should answer the problems individually. Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. Complete the table by using the given value for x. State whether the value makes a true or false inequality.

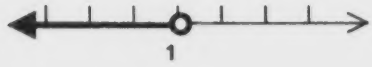
x	–3	–2	–1	0	1	2
$2x < 4$ True or False	True	True	True	True	True	False
$x < 1$ True or False	True	True	True	True	False	False
$5x + 3 < -2$ True or False	True	True	False	False	False	False
$8x + 3 < 3x - 2$ True or False	True	True	False	False	False	False
$3 + x > 3$ True or False	False	False	False	False	True	True
$3x + 5 > 2x + 5$ True or False	False	False	False	False	True	True

2. Graph the following inequalities on a number line.

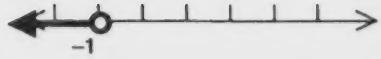
a. $2x < 4$



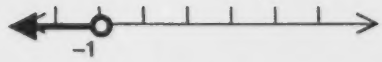
b. $x < 1$



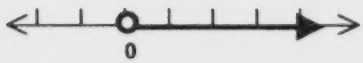
c. $5x + 3 < -2$



d. $8x + 3 < 3x - 2$



e. $3 + x > 3$



f. $3x + 5 > 2x + 5$

3. How does the table in problem 1 help you understand the graph of each inequality?

Answers will vary. For example, some students may notice equivalent inequalities, in which the graphs would be the same.

4. Write an equivalent inequality for each of the following:

a. $x < -1$ **Answers will vary. For example, $2x - 1 < -3$**

b. $x > 0$ **Answers will vary. For example, $-3x + 4 < 4$**

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LAB 4–A

Name _____

Partner _____

EXPRESSIONS AND EQUATIONS

Materials: Algebra Tiles, notebook paper

Work with a partner.

1. a. Use tiles to represent the expressions given. On a separate sheet of paper, draw a sketch of each representation.

   1) $2x + 3$

   2) $2(x + 3)$

   3) $3x + 2(x + 3)$

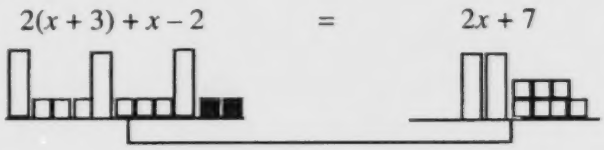
   4) $3x - 2(x + 3)$

   b. Simplify each expression in 1.a. Support your answer with the tile representations.

2. Doug used tiles to help him solve the equation $2(x + 3) + x - 2 = 2x + 7$.

   He said he thought of a balance scale.

   Explain how Doug may have used the tiles to solve the equation.



3. How could Doug solve the inequality $2(x + 3) + x - 2 > 2x + 7$ using the balance scale model?

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LAB 4–B

USING THE TI CALCULATOR

Name _____

Partner _____

Materials: TI-80, 82, or 83 calculator, notebook paper

Work with a partner. Record your group's answers on your own paper. Remember to take turns on the calculator.

1. Erin used a calculator to help her solve the equation $3x - 5x + 4 = 8$. This is what she did:

   1) She turned on the calculator, then hit the [Y=] key.

   2) Then she entered **$3x - 5x + 4$** where the blinking cursor was.

   3) Next she pressed the [2nd] key and then the [GRAPH] key. What happened?

   4) Erin also experimented with the blue arrow keys. What did these keys do?

   5) Describe how you think Erin was able to solve the equation.

If you have been pressing the keys on the calculator until now, let your partner use the calculator for the next problem.

2. How could you use Erin's method to solve $5(x + 4) - 1 = 9$? Explain thoroughly.

3. Patrick discovered the [QUIT] and the [TBLSET] commands on the calculator. How did he access them and what does each do?

It's time to switch again!

4. Chelsea tried solving $6x - 2(x + 2) = 6$ using Erin's method but was unable to find the solution. Kris suggested using [TBLSET] and then changing what ΔTBL equaled. Explain what she may have done.

Now it's time to switch for the last time.

5. Ryan made use of Y_1 and Y_2 in the [Y=] screen on the calculator to solve the following problems:

   Solve for f:

   a. $2f + 3 = 28 - 3f$
   b. $2f + 3 < 28 - 3f$

   What do you think Ryan did? How was he able to access the Y_2 column in the table?

LAB 4–C

EQUATION PROBLEM

Name _____

Group members _____

_____

_____

Date _____

1. You have several methods of solving equations. You can use a table, guess-and-check, algebra, diagrams, or a combination of these. Select a method and solve: $4x - 5 = 9 + 2x$.

   Solve it another way.

2. Simplify: $3(x + 2) - 4(5 - x)$.

   You can use the calculator to check your simplified expression. Pick a value for x and use your calculator to check. Show your displays below.

3. You've seen devices called *traffic counters* that count the vehicles passing along a street. You can recognize the devices as thin black cords stretched across a street or highway and connected to a "brain box" at the side of the road. The traffic counter is usually pressure activated. It registers a count each time a set of wheels (wheels on a single axle) rolls over the tube. A normal car registers two counts: one for the front wheels and one for the rear wheels. A truck with three axles (front wheels plus a double set of rear wheels) registers three counts. A large semi might register four or five counts.

   Suppose that during a one-hour period, a particular traffic counter registers 35 counts on a residential street where heavy trucks are prohibited. (That is, only two-axle vehicles like cars or three-axle vehicles like light trucks are allowed.) How many cars and light trucks passed over the traffic counter? (Show your work, and all solutions.)

4. Sometimes an equation can be used to show the relationship in the problem. In this problem, the equation would be $2c + 3t = 35$. Using any operations solve this equation for c. That is, your final equation should have c on the left and everything else on the right of the equal sign. This is called making c the ***subject*** of the equation.

   Describe what you did to write the equation in the form $c = ?$.

LAB 4–D

Name _____

Date _____

INEQUALITY LAB

1. Complete the table by using the given value for x. State whether the value makes a true or false inequality.

x	–3	–2	–1	0	1	2
$2x < 4$ True or False	True	True	True	True	True	False
$x < 1$ True or False						
$5x + 3 < -2$ True or False						
$8x + 3 < 3x - 2$ True or False						
$3 + x > 3$ True or False						
$3x + 5 > 2x + 5$ True or False						

2. Graph the following inequalities on a number line.

   a. $2x < 4$

   b. $x < 1$

   c. $5x + 3 < -2$

   d. $8x + 3 < 3x - 2$

   e. $3 + x > 3$

   f. $3x + 5 > 2x + 5$

3. How does the table in problem 1 help you understand the graph of each inequality?

4. Write an equivalent inequality for each of the following:

   a. $x < -1$

   b. $x > 0$

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Chapter 5 Graphing

Teacher Notes

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Student Worksheet Masters

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LAB 5–A NOTES

GRAPHING CALCULATOR SCAVENGER HUNT

Purpose:	Using graphing calculators Graphing Calculator Scavenger Hunt allows students to explore more functions of the graphing calculator.
Materials:	Graphing Calculator Scavenger Hunt sheet (one per student), graphing calculator
Directions:	Students work in pairs, but each person should complete a sheet. Allow about 10 minutes at the end of the period for students to discuss the keystrokes they used. Students may find more than one way to accomplish a task.

Record the keystroke(s) you used to accomplish each task.

Answers will vary depending on the graphing calculator used. For example, the keystrokes described below are for the TI–83+.

Task	Keystrokes
1. Adjust the contrast of the screen.	[2nd] followed by the up arrow key to darken [2nd] followed by the down arrow key to lighten
2. Evaluate $(-2)^3 - 4$.	Keys: [(], [(–)], [2], [)], [^], [3], [–], [4], [ENTER]
3. Graph $y = 2x - 1$.	[Y=], [2], [X,T,θ,n], [–], [1], [GRAPH]
4. Graph $y = -0.5x + 1$.	[Y=], [(–)], [.], [5], [X,T,θ,n], [+], [1], [GRAPH]
5. Show a table of values of x and y for $y = 2x - 1$.	[Y=], [2], [X,T,θ,n], [–], [1], [2nd], [GRAPH]
6. Find the point of intersection of $y = 2x - 1$ and $y = -0.5x + 1$.	[Y=], [2], [X,T,θ,n], [–], [1], [ENTER], [(–)], [.], [5], [X,T,θ,n], [+], [1], [2nd], [CALC], and select 5: intersect. Some students may use [GRAPH] or [2nd], [GRAPH] (using a table) to find the point of intersection.
7. Change the graph viewing window.	[WINDOW] and change the settings
8. Turn the calculator off.	[2nd], [ON]

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LAB 5–B NOTES

THE CARTESIAN COORDINATE SYSTEM

Purpose: Using graphing calculators

The Cartesian Coordinate System allows students to review and explore features of the Cartesian coordinate system through using a graphing calculator.

Materials: The Cartesian Coordinate System sheet (one per student), TI-graphing calculator, graph paper, notebook paper

Directions: Students work in pairs, but each person should answer the problems individually. Students may have difficulty understanding what a unit is.

Allow 5–10 minutes at the end of the period for students to discuss the settings they used. Some students should be able to generalize settings for some of the tasks.

Note: If programs are stored on the calculator, you may want students to clear only to defaults and clear others manually. If you have questions about clearing your memory, consult your calculator manual.

1. Clear the calculator memory. Press MEM and select the Reset settings until the display indicates that the memory is cleared.

2. Press ZOOM; select the Standard setting.

   a. Tick marks are used to indicate units on each axis. How many units are marked on the x-axis?

      20 units are marked on the *x*-axis.

   b. How many units are marked on the y-axis?

      20 units are marked on the *y*-axis.

3. Press WINDOW. Compare the values displayed with your answers to problems 2.a. and 2.b. and describe what you notice.

   Answers may vary. The default values are Xmin = –10, Xmax = 10, Ymin = –10 and Ymax = 10.

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LAB 5–B NOTES

4. a. Change XMIN to a negative integer greater than –10. What is your value for XMIN?

   Answers will vary.

   b. Change XMAX to a positive integer less than 10. What is your value for XMAX?

   Answers will vary.

   c. Change YMIN to a negative integer greater than –10. What is your value for YMIN?

   Answers will vary.

   d. Change YMAX to a positive integer less than 10. What is your value for YMAX?

   Answers will vary.

   e. Press GRAPH. How many units are marked on the *x*-axis?

   Answers will vary. The number of units on the *x*-axis should equal the sum of the absolute value of Xmin in 4.a. and the absolute value of Xmax in 4.b.

   f. How many units are marked on the *y*-axis?

   Answers will vary. The number of units on the *y*-axis equals the sum of the absolute value of Ymin in 4.c. and the absolute value of Ymax in 4.d.

   g. What effect do XMIN, XMAX, YMIN, and YMAX have on the graph display?

   Xmin sets the smallest unit on the *x*-axis, Xmax sets the largest unit on the *x*-axis, Ymin sets the smallest unit on the *y*-axis, and Ymax sets the largest unit on the *y*-axis.

5. On graph paper, draw the Cartesian coordinate plane and label the region for Quadrant I. Then press WINDOW and change the settings to display Quadrant I only on the graphing calculator.

   a. What is your value for XMIN? **Xmin = 0**

   b. What is your value for XMAX? **Xmax can be any positive value.**

   c. What is your value for YMIN? **Ymin = 0**

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LAB 5–B NOTES

5. d. What is your value for YMAX? **Ymax can be any positive value.**

   e. Are your answers to 5.a.–5.d. unique? Explain why or why not.

   Xmin and Ymin must both equal 0, and Xmax and Ymax can be any positive values. Answers may vary. Answers to 5.a. and 5.c. need not be 0.

6. On your graph, label the region for Quadrant II. Then press WINDOW and change the settings to display Quadrant II only on the graphing calculator.

   a. What is your value for XMIN? **Xmin can be any negative value.**

   b. What is your value for XMAX? **Xmax = 0**

   c. What is your value for YMIN? **Ymin = 0**

   d. What is your value for YMAX? **Ymax can be any positive value.**

   e. Are your answers to 6.a.-6.d. unique? Explain why or why not.

   Xmin can be any negative value, Ymax can be any positive value, and Xmax and Ymin must both equal 0.

7. On your graph, label the region for Quadrant III. Then press WINDOW and change the settings to display Quadrant III only on the graphing calculator.

   a. What is your value for XMIN? **Xmin can be any negative value.**

   b. What is your value for XMAX? **Xmax = 0**

   c. What is your value for YMIN? **Ymin can be any negative value.**

   d. What is your value for YMAX? **Ymax = 0**

   e. Are your answers to 7.a.-7.d. unique? Explain why or why not.

   Xmin and Ymin can be any negative values, and Xmax and Ymax must both equal 0.

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LAB 5–B NOTES

8. On your graph, label the region for Quadrant IV. Then press WINDOW and change the settings to display Quadrant IV only on the graphing calculator.

   a. What is your value for XMIN? **Xmin = 0**

   b. What is your value for XMAX? **Xmax can be any positive value.**

   c. What is your value for YMIN? **Ymin can be any negative value.**

   d. What is your value for YMAX? **Ymax = 0**

   e. Are your answers to 8.a.-8.d. unique? Explain why or why not.

      Xmin and Ymax must both equal 0, Xmax can be any positive value, and Ymin can be any negative value

9. Explain how to display each of the following using the WINDOW settings.

   a. Quadrants I and II only

      Xmin can be any negative value, Xmax can be any positive value, Ymin = 0, and Ymax can be any positive value

   b. Quadrants I and IV only

      Xmin = 0, Xmax can be any positive value, Ymin can be any negative value, and Ymax can be any positive value

   c. Quadrants III and IV only

      Xmin can be any negative value, Xmax can be any positive value, Ymin can be any negative value, and Ymax = 0

   d. Quadrants II and III only

      Xmin can be any negative value, Xmax = 0, Ymin can be any negative value, and Ymax can be any positive value

LAB 5–B NOTES

9. e. Quadrants II and IV only

It is not possible to show Quadrants II and IV without also displaying Quadrants I and III.

10. Press ZOOM and set the calculator to the Standard setting. Press WINDOW and change the values for XSCL and YSCL. Display the graph. What do you conclude that XSCL and YSCL are used for?

Xscl and Yscl change the increments of the tic marks on the *x*- and *y*-axes.

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LAB 5–C NOTES

FAMILIES OF LINES

Purpose:	Using graphing calculators Families of Lines allows students to explore and generalize how the graph of a line is related to its equation through using a graphing calculator.
Materials:	Families of Lines sheet (one per student), graphing calculator
Directions:	Students work in groups of two, three, or four, but each person should answer the problems. Allow about 10 minutes at the end of the period for students to discuss their solutions.

Using your graphing calculators, graph each family below. You may want to graph them simultaneously to notice features. For each family of lines, sketch the graphs of the four lines displayed on your graphing calculator. Then answer the questions given.

1. Parent graph: $y = x$
   $y = 2x$
   $y = 4x$
   $y = 8x$

   a. What do all lines in this family have in common? In what ways are they different from one another?

   Answers will vary. For example, all lines have *y*-intercept of 0 but have different steepness (slope).

   b. Write the equations of two more lines that will belong to this family. Explain.

   Answers will vary. For example, $y = 16x$ and $y = 32x$ where the coefficient of x is a power of 2.

   c. What kind of motion can be used to describe this family of lines?

   The lines in this family are rotations of each other.

LAB 5–C NOTES

2. Parent graph: $y = -2x$
   $y = -2x + 1$
   $y = -2x - 2$
   $y = -2x + 4$

   a. What do all lines in this family have in common? In what ways are they different from one another?

   Answers will vary. For example, all the lines have the same steepness (slope) but have different *y*-intercepts. The lines are parallel.

   b. Write the equation of another line that will belong to this family. Explain.

   Answers will vary. For example, $y = -2x - 8$ and $y = -2x + 16$.

   c. What kind of motion can be used to describe this family of lines?

   The lines in this family are translations of each other.

3. a. Set your graphing window to ZDecimal.

   b. Find equations of two lines that will go through the origin and be perpendicular to each other. Describe how you went about finding your two equations.

   Answers will vary. The slopes of the lines should be opposite reciprocals, and the *y*-intercept of both lines should be 0.

   c. Find an equation of a line that will be perpendicular to $y = \frac{2}{3}x$. Explain how you got your equation.

   Answers will vary. The coefficient of *x* must be $-\frac{3}{2}$.

4. a. Given the equation $y = 4x + 2$, create a table of four pairs of x- and y-values.

   Answers will vary. For example,

x	–1	0	1	2
y	–2	2	6	10

   b. Predict what the graph will look like. **Answers will vary.**

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4. c. Graph $y = 4x + 2$. Was your prediction correct? Explain.

Answers will vary depending on response to 4.b.

d. What would be the effect on the table and the graph if you changed the equation to $y = 2x + 2$?

Answers will vary. For example, values for *x* can remain the same, but the corresponding values for *y* will change. The line for $y = 2x + 2$ has the same intercept as the line for $y = 4x + 2$ but is not as steep.

e. What if you had changed the equation to $y = 4x - 2$ instead?

Answers will vary. For example, values for *x* can remain the same, but the corresponding values for *y* will change. The lines for $y = 4x + 2$ and $y = 4x - 2$ are parallel.

5. Since the calculator only accepts equations in the form $y =$ _____, Ryan wondered how he could graph $3x - 5y = 15$ on the calculator. What would you tell him?

Answers will vary. For example, Ryan could use algebraic notation and write the equation as shown:

$$\begin{aligned} 3x - 5y &= 15 \\ -5y &= -3x + 15 \qquad A_{-3x} \\ y &= \tfrac{3}{5}x - 3 \qquad D_5 \end{aligned}$$

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LAB 5–D NOTES

A KNOTTY PROBLEM

Purpose: Extending concept development

A Knotty Problem allows students to gather data that generate a linear pattern and to write an equation corresponding to their line.

Materials: A Knotty Problem sheet (one per student), rope (about 1 yard), a meter stick or metric tape, graph paper, notebook paper, ruler

Directions: Place students in groups of four and distribute materials. Students should work as a group, but each person is responsible for answering the problems individually. Tell students they will be tying one knot at a time in the rope. Demonstrate the knot. Pause and ask them to respond to question 1. Go over procedures for problem 2 with the class before groups begin working on the lab.

Allow about 10 minutes at the end of the period for students to discuss their findings.

1. You will collect data showing how the length of a rope changes as you tie more knots in it. Before your group begins, discuss what you expect to find out. Write your group's prediction about what you think will happen.

   Answers will vary. Students should include a reason for their predictions.

2. Measure the length of the rope before you tie any knots and record the length in the table below. Tie one knot in the rope, measure the new length, and record it in the data table. Continue tying knots in the rope, measuring and recording data until you have nine knots.

   Data will vary.

Number of Knots	Length of Rope (cm)
0	
1	
2	
3	
4	

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LAB 5–D NOTES

3. Plot the data on graph paper. Be sure to label your axes clearly.

   The points graphed should correspond to the data in the table.

4. What do you conclude from your graph? Why do you think you got these results?

   Answers will vary. Students should be able to support their conclusions from their graphs.

5. Write an equation for your graph.

   Answers will vary. The equation should correspond to the graph. If the equation is written in slope-intercept form, debriefing may include explaining what the *y*-intercept represents, what affects the slope, and why the slope is negative.

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LAB 5–E NOTES

WATER SPOTS

Purpose: Extending concept development

Water Spots allows students to gather data that generate a linear pattern and to write an equation corresponding to their line.

Materials: Water Spots sheet (one per student), two sheets of paper towel, one medicine dropper, a cup of water, a ruler, graph paper, paper

Directions: Place students in groups of four and distribute materials. Students should work as a group, but each person should answer the problems individually.

Give each group a small cupful of colored water. Advise students to outline the spot then wait to be sure the spot does not spread beyond the outline before measuring. Allow about 10 minutes at the end of the period for students to discuss their findings.

1. Lay the two paper towels on top of each other. Drop one drop of water onto the towels. (Avoid dropping water on any folds or creases.) As soon as the spot has finished spreading, measure its diameter. Record your data in the table below. Repeat this process for 2, 3, and 4 drops.

Data will vary.

Number of drops in the spot	Diameter of the spot
1	
2	
3	
4	

2. Plot your data on graph paper.

The points graphed should correspond to the data in the table.

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LAB 5–E NOTES

3. What if you used 6 drops of water? Predict what the diameter of the spot would be. Explain how you got your prediction.

Answers will vary. An explanation must be included with the prediction.

4. What if you used 10 drops of water? Predict what the diameter of the spot would be. Explain how you got your prediction.

Answers will vary. An explanation must be included with the prediction.

5. About how many drops would be needed to form a circle with a diameter of 11 inches? Explain how you solved this.

Answers will vary. An explanation must be included with the prediction.

6. Draw a straight line that you feel best describes the data you plotted in problem 2. Explain why you chose that line.

Answers will vary. An explanation must be included for the line.

7. Find a rule that describes your line. Explain how you arrived at your rule.

Answers will vary. Equation and graph should correspond.

EXTENSIONS: Select at least one of the following questions. Write your response with a justification.

- What would happen if you were to use a straw instead of a medicine dropper?
- What would happen if you used a different liquid?
- What would happen if you used a different brand of paper towel?

Answers will vary. Students should include the reasoning for their predictions.

LAB 5–F NOTES

MISSION POSSIBLE: WALK, DON'T RUN

Purpose:	Using graphing calculators Mission Possible: Walk, Don't Run allows students to interpret a graph and translate it into a walking pattern.
Materials:	Mission Possible: Walk, Don't Run sheet (one per student), TI-82 or 83 with a unit-to-unit link cable, Calculator-Based Laboratory (CBL) unit, Motion Detector, paper
Directions:	Place students in groups of four and distribute materials. Students should work as a group, but each person should answer the problems individually. Go over the procedures and demonstrate how to use the motion detector. Allow about 10 minutes at the end of the period for students to discuss their findings.

1. Choose a Starter and a Walker for graph 1. The Starter will control the keys on the TI calculator. The Walker will carry out the walking pattern planned by your group.

2. Discuss and plan out the walking pattern to be used by Walker 1.

   Plans will vary. Students should include reasons for their plans.

3. Make sure the CBL is turned on. Start the HIKER program. The program will display a message, "PRESS ENTER TO START GRAPH." When the Starter and the Walker are ready, the Starter will press ENTER and upon hearing a clicking sound, tell the Walker to start walking.

4. The graphing calculator will display a graph of distance (y-axis) against time (x-axis) as the data are collected. If the Walker has produced a reasonably good graph, write up your group's walking plan. If modifications need to be made in the walking pattern, repeat steps 2 and 3 until your group is satisfied.

   Plans will vary. If modifications to the original plan are made, an explanation should be given.

5. Assign a new Walker and Starter for graph 2. Repeat steps 2–4 above.

6. Repeat the above procedure for graphs 3 and 4.

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LAB 5–G NOTES

BOUNCING BALL

Purpose: Extending concept development

Bouncing Ball allows students to gather data that generate a linear pattern and to write an equation corresponding to their line.

Materials: Bouncing Ball sheet (one per student), SuperBall, tape measure, masking tape, graphing calculator, graph paper

Directions: Students work in pairs but each person should answer the problems individually. Students should alternate in their roles of ball bouncer and recorder.

Allow about 10 minutes at the end of the period for students to discuss their findings.

1. On the wall designated for your pair, put a tape marker at the 1-foot, 2-feet, 3-feet, 4-feet, and 5-feet marks.

2. Drop the ball from the 1-foot mark. Make sure that the other person in your pair is ready to watch. Record the height the ball went to after its first bounce. Drop the ball again from the 1-foot mark. Record the height after its first bounce. Repeat two more times. You should have four heights recorded in the table below. Repeat this same procedure for 2 feet, 3 feet, 4 feet, and 5 feet.

Data will vary.

x	*y*
1'	
1'	
1'	
1'	
2'	
2'	
2'	

LAB 5–G NOTES

3. Graph your points on graph paper.

   The points graphed should correspond to the data in the table.

4. Draw a line that seems to hit as many points as possible. (You may notice that your line is between some points.) This is called *the line of best fit.*

   The line drawn should appear to be the line of best fit for the points graphed.

5. Find a rule for the line of best fit. Describe how you found the rule.

   Answers will vary. If an equation is given, the equation and graph of the line should correspond to the data and the graph.

LAB 5–A

Name _____

Date _____

GRAPHING CALCULATOR SCAVENGER HUNT

Record the keystroke(s) you used to accomplish each task.

Task	Keystrokes
1. Adjust the contrast of the screen.	
2. Evaluate $(-2)^3 - 4$.	
3. Graph $y = 2x - 1$.	
4. Graph $y = -0.5x + 1$.	
5. Show a table of values of x and y for $y = 2x - 1$.	
6. Find the point of intersection of $y = 2x - 1$ and $y = -0.5x + 1$.	
7. Change the graph viewing window.	
8. Turn the calculator off.	

Make and model of your graphing calculator _____

LAB 5–B

THE CARTESIAN COORDINATE SYSTEM

Name _____

Partner _____

Date _____

Materials: TI-graphing calculator, graph paper, notebook paper

1. Clear the calculator memory. Press MEM and select the Reset settings until the display indicates that the memory is cleared.

2. Press ZOOM; select the Standard setting.

   a. Tick marks are used to indicate units on each axis. How many units are marked on the x-axis?

   b. How many units are marked on the y-axis?

3. Press WINDOW. Compare the values displayed with your answers to problems 2.a. and 2.b. and describe what you notice.

4. a. Change XMIN to a negative integer greater than –10. What is your value for XMIN?

   b. Change XMAX to a positive integer less than 10. What is your value for XMAX?

   c. Change YMIN to a negative integer greater than –10. What is your value for YMIN?

   d. Change YMAX to a positive integer less than 10. What is your value for YMAX?

   e. Press GRAPH. How many units are marked on the x-axis?

   f. How many units are marked on the y-axis?

   g. What effect do XMIN, XMAX, YMIN, and YMAX have on the graph display?

5. On graph paper, draw the Cartesian coordinate plane and label the region for Quadrant I. Then press WINDOW and change the settings to display Quadrant I only on the graphing calculator.

   a. What is your value for XMIN?

   b. What is your value for XMAX?

   c. What is your value for YMIN?

   d. What is your value for YMAX?

   e. Are your answers to 5.a.–5.d. unique? Explain why or why not.

6. On your graph, label the region for Quadrant II. Then press WINDOW and change the settings to display Quadrant II only on the graphing calculator.

   a. What is your value for XMIN?

   b. What is your value for XMAX?

   c. What is your value for YMIN?

   d. What is your value for YMAX?

   e. Are your answers to 6.a.–6.d. unique? Explain why or why not.

7. On your graph, label the region for Quadrant III. Then press WINDOW and change the settings to display Quadrant III only on the graphing calculator.

   a. What is your value for XMIN?

   b. What is your value for XMAX?

   c. What is your value for YMIN?

   d. What is your value for YMAX?

   e. Are your answers to 7.a.–7.d. unique? Explain why or why not.

8. On your graph, label the region for Quadrant IV. Then press WINDOW and change the settings to display Quadrant IV only on the graphing calculator.

   a. What is your value for XMIN?

   b. What is your value for XMAX?

   c. What is your value for YMIN?

   d. What is your value for YMAX?

   e. Are your answers to 8.a.–8.d. unique? Explain why or why not.

9. Explain how to display each of the following using the WINDOW settings.

   a. Quadrants I and II only

   b. Quadrants I and IV only

   c. Quadrants III and IV only

   d. Quadrants II and III only

   e. Quadrants II and IV only

10. Press ZOOM and set the calculator to the Standard setting. Press WINDOW and change the values for XSCL and YSCL. Display the graph. What do you conclude that XSCL and YSCL are used for?

LAB 5–C

FAMILIES OF LINES

Name _____

Group members _____

_____

Date _____

Materials: TI-graphing calculator, notebook paper

Using your graphing calculators, graph each family below. You may want to graph them simultaneously to notice features. For each family of lines, sketch the graphs of the four lines displayed on your graphing calculator. Then answer the questions given.

1. Parent graph: $y = x$
   $y = 2x$
   $y = 4x$
   $y = 8x$

   a. What do all lines in this family have in common? In what ways are they different from one another?
   b. Write the equations of two more lines that will belong to this family. Explain.
   c. What kind of motion can be used to describe this family of lines?

2. Parent graph: $y = -2x$
   $y = -2x + 1$
   $y = -2x - 2$
   $y = -2x + 4$

   a. What do all lines in this family have in common? In what ways are they different from one another?
   b. Write the equation of another line that will belong to this family. Explain.
   c. What kind of motion can be used to describe this family of lines?

3. a. Set your graphing window to ZDecimal.
   b. Find equations of two lines that will go through the origin and be perpendicular to each other. Describe how you went about finding your two equations.
   c. Find an equation of a line that will be perpendicular to $y = \frac{2}{3}x$. Explain how you got your equation.

4. a. Given the equation $y = 4x + 2$, create a table of four pairs of x- and y-values.
   b. Predict what the graph will look like.
   c. Graph $y = 4x + 2$. Was your prediction correct? Explain.
   d. What would be the effect on the table and the graph if you changed the equation to $y = 2x + 2$?
   e. What if you had changed the equation to $y = 4x - 2$ instead?

5. Since the calculator only accepts equations in the form $y =$ _____, Ryan wondered how he could graph $3x - 5y = 15$ on the calculator. What would you tell him?

LAB 5–D

A KNOTTY PROBLEM

Name _____

Group members _____

_____

_____

Date _____

Materials: Rope (about 1 yard), a meter stick or metric tape, graph paper, notebook paper, ruler

1. You will collect data showing how the length of a rope changes as you tie more knots in it. Before your group begins, discuss what you expect to find out. Write your group's prediction about what you think will happen.

2. Measure the length of the rope before you tie any knots and record the length in the table below. Tie one knot in the rope, measure the new length, and record it in the data table. Continue tying knots in the rope, measuring and recording data until you have nine knots.

Number of Knots	Length of Rope (cm)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

3. Plot the data on graph paper. Be sure to label your axes clearly.

4. What do you conclude from your graph? Why do you think you got these results?

5. Write an equation for your graph.

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LAB 5–E

WATER SPOTS

Name _____

Group members _____

_____

_____

Date _____

Materials: Two sheets of paper towel, one medicine dropper, a cup of water, a ruler, and graph paper

1. Lay the two paper towels on top of each other. Drop one drop of water onto the towels. (Avoid dropping water on any folds or creases.) As soon as the spot has finished spreading, measure its diameter. Record your data in the table below. Repeat this process for 2, 3, and 4 drops.

Number of drops in the spot	Diameter of the spot
1	
2	
3	
4	

2. Plot your data on graph paper.

3. What if you used 6 drops of water? Predict what the diameter of the spot would be. Explain how you got your prediction.

4. What if you used 10 drops of water? Predict what the diameter of the spot would be. Explain how you got your prediction.

5. About how many drops would be needed to form a circle with a diameter of 11 inches? Explain how you solved this.

6. Draw a straight line that you feel best describes the data you plotted in problem 2. Explain why you chose that line.

7. Find a rule that describes your line. Explain how you arrived at your rule.

EXTENSIONS: Select at least one of the following questions. Write your response with a justification.

- What would happen if you were to use a straw instead of a medicine dropper?
- What would happen if you used a different liquid?
- What would happen if you used a different brand of paper towel?

LAB 5–F

Name _____

MISSION POSSIBLE: WALK, DON'T RUN

Group members _____

_____

_____

Date _____

Materials: TI-82 or 83 with a unit-to-unit link cable, Calculator-Based Laboratory (CBL) unit, Motion Detector, paper

Group goal: To have the group plan and carry out a walk so that the graphs of the function of the walker's distance from the motion detector with respect to time will form a line with a slope greater than 1.

Procedure: A motion detector has been set up so that it will detect the movement of a student walking away from the detector. To ensure that only the actions of the walker will be detected by the motion detector, no one but the walker should be in the walkway. The walker must walk perpendicular to the detector. (Note: The motion detector cannot detect objects closer than 1.5 feet of the detector.) The detector takes measurements in feet every 0.1 seconds for 6 seconds.

1. Choose a Starter and a Walker for graph 1. The Starter will control the keys on the TI calculator. The Walker will carry out the walking pattern planned by your group.

2. Discuss and plan out the walking pattern to be used by Walker 1.

3. Make sure the CBL is turned on. Start the HIKER program. The program will display a message, "PRESS ENTER TO START GRAPH." When the Starter and the Walker are ready, the Starter will press ENTER and upon hearing a clicking sound, tell the Walker to start walking.

4. The graphing calculator will display a graph of distance (y-axis) against time (x-axis) as the data are collected. If the Walker has produced a reasonably good graph, write up your group's walking plan. If modifications need to be made in the walking pattern, repeat steps 2 and 3 until your group is satisfied.

5. Assign a new Walker and Starter for graph 2. Repeat steps 2–4 above.

6. Repeat the above procedure for graphs 3 and 4.

LAB 5–G

BOUNCING BALL

Name _____

Partner _____

Date _____

Materials: SuperBall, tape measure, masking tape, graphing calculator, graph paper

1. On the wall designated for your pair, put a tape marker at the 1-foot, 2-feet, 3-feet, 4-feet, and 5-feet marks.

2. Drop the ball from the 1-foot mark. Make sure that the other person in your pair is ready to watch. Record the height the ball went to after its first bounce. Drop the ball again from the 1-foot mark. Record the height after its first bounce. Repeat two more times. You should have four heights recorded in the table below. Repeat this same procedure for 2-feet, 3-feet, 4-feet, and 5-feet.

x	y
1'	
1'	
1'	
1'	
2'	
2'	
2'	
2'	
3'	
3'	
3'	
3'	
4'	
4'	
4'	
4'	
5'	
5'	
5'	
5'	

3. Graph your points on graph paper.

4. Draw a line that seems to hit as many points as possible. (You may notice that your line is between some points.) This is called *the line of best fit*.

5. Find a rule for the line of best fit. Describe how you found the rule.

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Chapter 6 Systems of Equations and Inequalities

Teacher Notes

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Student Worksheet Masters

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LAB 6–A NOTES

GRAPHING LINEAR INEQUALITIES

Purpose: Using graphing calculators

Graphing Linear Inequalities allows students to compare graphs of equations and inequalities using a graphing calculator.

Materials: Graphing Linear Inequalities sheet (one per student), TI-80 graphing calculator, graph paper

Directions: Students work in pairs, but each person should answer the problems individually. Remind students to take turns using the graphing calculators if there are not enough to go around.

Allow 5–10 minutes at the end of the period for students to discuss the settings they used.

1. a. Write three ordered pairs that are solutions to the equation $y = 3x + 1$.

Answers will vary. For example, (–2, –5), (0, 1), (3, 10)

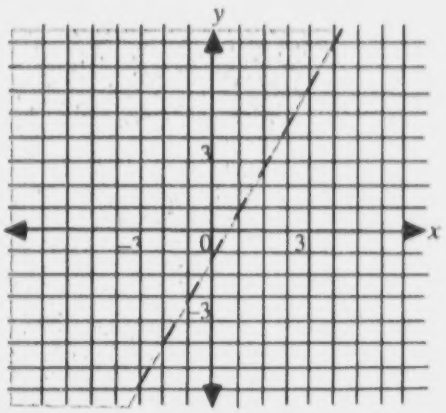
b. Write three ordered pairs that are solutions to the inequality $y > 3x + 1$.

Answers will vary. For example, (–2, –4), (0, 2), (3, 11)

c. Write three ordered pairs that are solutions to the inequality $y < 3x + 1$.

Answers will vary. For example, (–2, –6), (0, 0), (3, 9) or (–2, –8), (0, –15), (4, 12)

d. On the calculator, press DRAW (2nd and PRGM) and select Shade_Y>. Type in $3x + 1$ and press ENTER. Sketch the display on the calculator onto graph paper. [**Note**: On higher-numbered models, press Y= and enter $y = 3x + 1$. Press DRAW, then select Shade and input the entry Shade (Y_1, 10). To display Y_1, press VARS; select Y-VARS, then select Function followed by Y_1.]



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LAB 6–A NOTES

1. e. Describe where the points for the ordered pairs in 1.a. are on the display: on the line $y = 3x + 1$, on the shaded side of the line, or on the unshaded side of the line.

The points are on the line $y = 3x + 1$.

f. Describe where the points for the ordered pairs in 1.b. are on the display.

The points are on the shaded side of the line $y = 3x + 1$.

g. Describe where the points for the ordered pairs in 1.c. are on the display.

The points are on the unshaded side of the line $y = 3x + 1$.

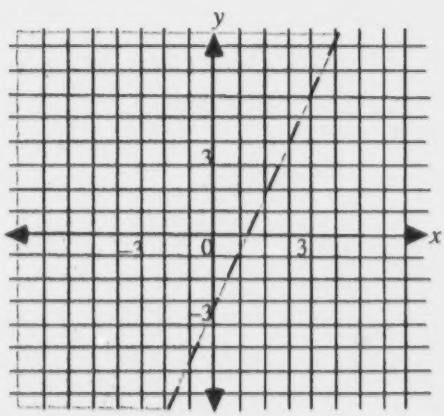
h. How do you think the graphs of $y > 3x + 1$ and $y \geq 3x + 1$ compare?

The graph of $y \geq 3x + 1$ includes the same points as the graph of $y > 3x + 1$ in addition to the points on the line $y = 3x + 1$.

2. a. Write three ordered pairs that are solutions to the inequality $y < 2x - 3$.

Answers will vary. For example, (–3, –10), (0, 4), (2, 0)

b. On the calculator, press DRAW (2nd and PRGM) and select Shade_Y<. Type in $2x - 3$ and press ENTER. Sketch the display on the calculator onto graph paper. [**Note**: On higher-numbered models, press Y= and enter $y = 2x - 3$. Press DRAW, then select Shade and input the entry Shade $(-10, Y_1)$. To display Y_1, press VARS; select Y-VARS, then select Function followed by Y_1.]

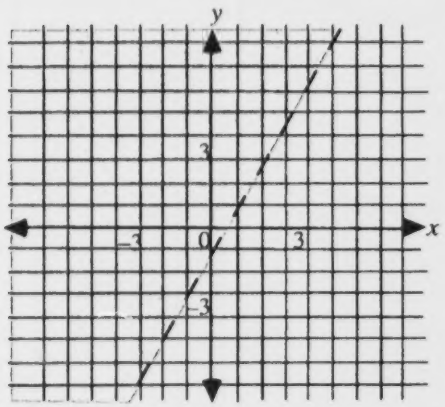


c. Describe where the points for the ordered pairs in 2.a. are on the display.

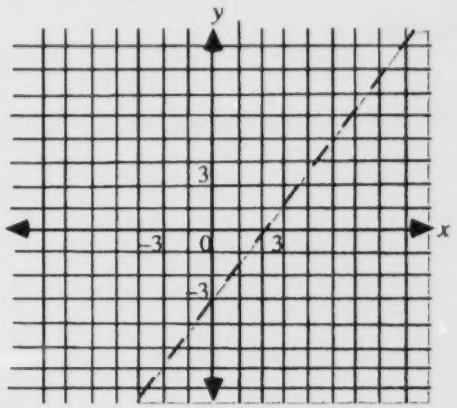
The points are in the shaded side of the line $y = 2x - 3$.

LAB 6–A NOTES

3. Graph $y > 2x - 1$. Use the graphing calculator to verify your graph.



4. Write an inequality for the graph shown below. Use the graphing calculator to verify your inequality.



$y < x - 3$

5. How is graphing an inequality like or unlike graphing an equation? Use examples to support your answer.

Answers will vary. Students may describe a process that uses the linear representations to find the "boundary."

LAB 6–B NOTES

IF THE SHOE FITS

Purpose: Extending concept development

If the Shoe Fits allows students to use personal data to generate linear patterns and to write equations corresponding to their lines. They can then make predictions based on the linear patterns.

Materials: If the Shoe Fits sheet (one per student)

Directions: Place students in groups of four. Before this lab, have students measure their height and record their shoe size. Post the class data. Students work in groups, but each person should answer the problems individually.

Allow 5–10 minutes at the end of the period for students to discuss their predictions and conjectures. Encourage students to make generalizations from the class data.

1. Display the class data at the front of the room. From the data given, what relationship, if any, appears to exist between a person's height and shoe size?

   Answers will vary but should reflect the class data.

2. Graph the data to confirm or reject your observation. In your group, decide how best to graph the data. For example, should you distinguish male and female data points? Why or why not?

   The points graphed should correspond to the class data.

3. For each set of data points you graphed, draw the line of best fit. Explain how your group decided where to draw the line of best fit. Find an equation of each line you drew.

   Answers will vary and should include an explanation. Equations should correspond to the respective lines.

4. If Samantha is 5 feet, 1 inch tall, what would you estimate her shoe size to be? Explain how you got your estimate.

   Answers will vary and should include an explanation.

5. If Brad is 6 feet, 3 inches tall, what would you estimate his shoe size to be? Explain.

Answers will vary and should include an explanation.

6. The police found a deep footprint for a size 9 shoe shaped like this:

What conjectures might they make?

Answers will vary and should include an explanation.

LAB 6–C NOTES

BASKETS OF COCONUTS

Purpose: Problem solving

Students may want to use counters or other small manipulatives to help them solve the problem.

Materials: Baskets of Coconuts sheet (one per student), notebook paper, two-color counters (optional) and calculator (optional)

Directions: Place students in groups of four. They work in groups, but each person should answer the problems individually.

Allow students 25–30 minutes to complete. Monitor groups and advise them individually as they start problem 2 that they cannot use guess-and-test. Before the period ends, have them share their methods.

1. Five baskets contain coconuts. The first and second baskets together have a total of 52 coconuts. The second and third baskets have 43 coconuts. The third and fourth baskets have 34 coconuts. The fourth and fifth baskets have 30 coconuts. And the first and fifth baskets have 47 coconuts. How many coconuts are in each basket?

   The first basket has 30 coconuts, the second has 22, the third has 21, the fourth has 13, and the fifth has 17.

   Students who use guess-and-test should explain their reasoning as they adjust their guesses to find the solution. Some students may use systems of equations.

2. Solve the problem another way, excluding the guess-and-test method. Describe how this method is similar to or different from the method you used in problem 1.

   Students may have used guess-and-test for problem 1 and then a different method for problem 2. However, they should not use guess-and-test for problem 2 even if they did not use it for problem 1.

LAB 6–D NOTES

HOW MUCH DO I HAVE?

Purpose: Problem solving

How Much Do I Have? allows students to look for patterns related to divisibility.

Materials: How Much Do I Have? sheet (one per student), calculator

Directions: Place students in groups of four. They work in groups, but each person should answer the problems individually.

Allow students 25–30 minutes to complete. Students may use algebraic equations but not realize that many different variables must be used. Before the period ends, have them share their methods.

1. "Wow," said Malika. "I have a special amount of money in my savings account."

   "How much do you have?" asked Caesar.

   "Well," said Malika, "I can give you these clues. Let's see if you can figure out this special amount. I will tell you that it is less than $3,000.

   1. Divided by 10, the remainder is 9.
   2. Divided by 9, the remainder is 8.
   3. Divided by 8, the remainder is 7.
   4. The pattern continues down to a remainder of 1 when it is divided by 2."

   Malika has $2519.
   Students should realize that most calculators do not express division answers in quotient-and-remainder form.

2. What patterns did you use to solve this problem? Or what algebraic ideas helped you? Explain.

   Answers will vary. For example, from clue 1 students may conclude that the ones digits must be 9.

LAB 6–E NOTES

ON THE BALL

Purpose:	Problem solving
	On the Ball allows students to solve a problem involving inequalities.
Materials:	On the Ball sheet (one per student), graph paper, ruler
Directions:	Students work in pairs, but each person should answer the problems individually.
	Allow 5–10 minutes at the end of the period for students to discuss their solutions.

The Ball Factory has two machines that make basketballs. Machine A produces 30 basketballs per hour and costs $20 an hour to operate. Machine B produces 50 basketballs per hour and costs $30 an hour to operate. The factory must produce at least 650 basketballs in a 10-hour day.

1. What is the maximum number of hours each machine can be used? Explain your reasoning.

   Students may interpret this question differently. Some may think of operating both machines for 10 hours, thus producing 800 basketballs. Others may consider the number of hours needed to produce at least 650 basketballs and give answers such as "Machine A runs 10 hours and Machine B runs at least 7 hours."

2. To minimize operating costs, how many hours a day should each machine be used? Describe your method and justify your solution.

   Machine A should run 5 hours per day, and Machine B should run 10 hours per day. Methods will vary. Students may make a table of values. This problem can also be solved using linear programming by graphing the inequalities $30x + 50y \geq 650$, $0 \leq x \leq 10$, and $0 \leq y \leq 10$ where x is the number of hours Machine A runs and y is the number of hours Machine B runs.

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LAB 6–A

GRAPHING LINEAR INEQUALITIES

Name _____

Partner _____

Date _____

Materials: TI-80 graphing calculator, graph paper

Before you start, clear the calculator's memory. Press MEM and select the Reset settings until the display indicates that the memory is cleared.

1. a. Write three ordered pairs that are solutions to the equation $y = 3x + 1$.

   b. Write three ordered pairs that are solutions to the inequality $y > 3x + 1$.

   c. Write three ordered pairs that are solutions to the inequality $y < 3x + 1$.

   d. On the calculator, press DRAW (2nd and PRGM) and select Shade_Y>. Type in $3x + 1$ and press ENTER. Sketch the display on the calculator onto graph paper. [**Note**: On higher-numbered models, press Y= and enter $y = 3x + 1$. Press DRAW, then select Shade and input the entry Shade (Y_1, 10). To display Y_1, press VARS; select Y-VARS, then select Function followed by Y_1.]

   e. Describe where the points for the ordered pairs in 1.a. are on the display: on the line $y = 3x + 1$, on the shaded side of the line, or on the unshaded side of the line.

   f. Describe where the points for the ordered pairs in 1.b. are on the display.

   g. Describe where the points for the ordered pairs in 1.c. are on the display.

BEST COPY AVAILABLE

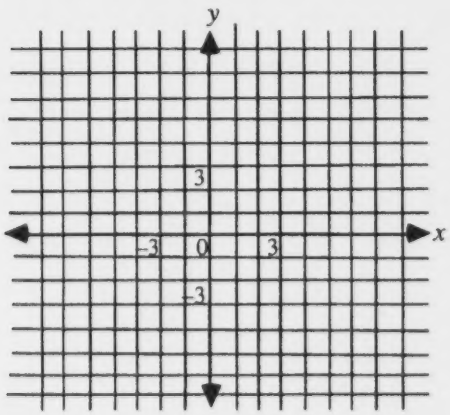
1. h. How do you think the graphs of $y > 3x + 1$ and $y \geq 3x + 1$ compare?

2. a. Write three ordered pairs that are solutions to the inequality $y < 2x - 3$.

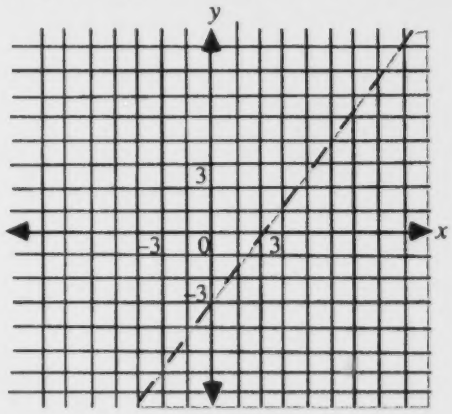
   b. On the calculator, press DRAW (2nd and PRGM) and select Shade_Y<. Type in $2x - 3$ and press ENTER. Sketch the display on the calculator onto graph paper. [**Note**: On higher numbered models, press Y= and enter $y = 2x - 3$. Press DRAW then select Shade and input the entry Shade $(-10, Y_1)$. To display Y_1, press VARS; select Y-VARS, then select Function followed by Y_1.]

   c. Describe where the points for the ordered pairs in 2.a. are on the display.

3. Graph $y > 2x - 1$. Use the graphing calculator to verify your graph.



4. Write an inequality for the graph shown below. Use the graphing calculator to verify your inequality.



5. How is graphing an inequality like or unlike graphing an equation? Use examples to support your answer.

LAB 6–B

IF THE SHOE FITS

Name _____

Group members _____

_____

_____

Date _____

Work as a group. Write your responses on a separate sheet of paper.

1. Display the class data at the front of the room. From the data given, what relationship, if any, appears to exist between a person's height and shoe size?

2. Graph the data to confirm or reject your observation. In your group, decide how best to graph the data. For example, should you distinguish male and female data points? Why or why not?

3. For each set of data points you graphed, draw the line of best fit. Explain how your group decided where to draw the line of best fit. Find an equation of each line you drew.

4. If Samantha is 5 feet, 1 inch tall, what would you estimate her shoe size to be? Explain how you got your estimate.

5. If Brad is 6 feet, 3 inches tall, what would you estimate his shoe size to be? Explain.

6. The police found a deep footprint for a size 9 shoe shaped like this:

What conjectures might they make?

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LAB 6–C

BASKET OF COCONUTS

Name _____

Group members _____

_____

_____

Date _____

Materials: Notebook paper, two-color counters (optional), and calculator (optional)

1. Work with your group to solve this problem. You may solve it using any strategy.

   Five baskets contain coconuts. The first and second baskets together have a total of 52 coconuts. The second and third baskets have 43 coconuts. The third and fourth baskets have 34 coconuts. The fourth and fifth baskets have 30 coconuts. And the first and fifth baskets have 47 coconuts. How many coconuts are in each basket?

2. Solve the problem another way, excluding the guess-and-test method. Describe how this method is similar to or different from the method you used in problem 1.

LAB 6–D

HOW MUCH DO I HAVE?

Name _____

Group members _____

_____

_____

Date _____

1. "Wow," said Malika. "I have a special amount of money in my savings account."

   "How much do you have?" asked Caesar.

   "Well," said Malika, "I can give you these clues. Let's see if you can figure out this special amount. I will tell you that it is less than $3,000.

   1. Divided by 10, the remainder is 9.
   2. Divided by 9, the remainder is 8.
   3. Divided by 8, the remainder is 7.
   4. The pattern continues down to a remainder of 1 when it is divided by 2."

   Describe your solution process. How much does Malika have?

2. What patterns did you use to solve this problem? Or what algebraic ideas helped you? Explain.

LAB 6–E

ON THE BALL

Name _____

Partner _____

Date _____

Materials: Graph paper, ruler

The Ball Factory has two machines that make basketballs. Machine A produces 30 basketballs per hour and costs $20 an hour to operate. Machine B produces 50 basketballs per hour and costs $30 an hour to operate. The factory must produce at least 650 basketballs in a 10-hour day.

1. What is the maximum number of hours each machine can be used? Explain your reasoning.

2. To minimize operating costs, how many hours a day should each machine be used? Describe your method and justify your solution.

Chapter 7 Polynomials

Teacher Notes

Lab 7–A	Power Bingo 1	7–1
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Student Worksheet Masters

Lab 7–A	Power Bingo 1	7–5
Lab 7–B	Powers of Binomials	7–7

LAB 7–A NOTES

POWER BINGO 1

Purpose: Extending concept development

Power Bingo 1 uses a game format that lets students practice finding monomial products.

Materials: Power Bingo 1 sheet (one per student), notebook paper, sack containing four each of transparency pieces labeled $3x$, $3y$, x^2, and y^2

Directions: Students may find the monomial products as homework before playing Power Bingo 1.

Have students share the products they find. When they agree that all products have been found, have them fill in *one* game square on the Power Bingo Game sheet. For the first game, you may want them to enter each product only once in their game square.

Make four copies of each monomial factor on transparency pieces and put them in a sack. From the sack, randomly select, announce, and display three transparency pieces on the overhead for students to find the product. Keep a record of the products to verify winners' Bingos.

Variations may include:

- filling the second game square allowing duplicate products entered in the cells. However, each product can be covered only one at a time.
- pulling three monomials at a time but not replacing them in the sack until three monomials can no longer be pulled.
- winning with four corners or a 2-by-2 square anywhere on the game square.

LAB 7–B NOTES

POWERS OF BINOMIALS

Purpose:	Extending concept development Powers of Binomials extends students' experiences with finding products of binomials to powers greater than 2.
Materials:	Powers of Binomials sheet (one per student)
Directions:	Place students in groups of four. Students work in groups, but each person should answer the problems individually. Allow 5–10 minutes at the end of the period for students to discuss patterns they noticed.

1. Have each member of your group select one of the problems below and find the cube of the binomial. Share your results and write the products in descending powers of the variable.

   a. $(x + y)^3$ $x^3 + 3x^2y + 3xy^2 + y^3$

   b. $(p + 2)^3$ $p^3 + 6p^2 + 12p + 8$

   c. $(3 + m)^3$ $m^3 + 9m^2 + 27m + 27$

   d. $(c - d)^3$ $c^3 - 3c^2d + 3cd^2 - d^3$

2. The product obtained by cubing a binomial forms a pattern. Compare your results from problem 1 with your group members' results and describe a pattern you notice.

   Answers will vary.

   For example, the cubes of both terms of the binomial are terms of the product, the product has 4 terms, or the coefficients of the second and third terms are multiples of 3.

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LAB 7–B NOTES

3. Use the pattern you described in problem 2 to find the cube of each binomial below:

   a. $(m-1)^3$

   $m^3 - 3m^2 + 3m - 1$

   b. $(j+2k)^3$

   $j^3 + 6j^2k + 12jk^2 + 8k^3$

4. The product obtained by taking the fourth power of a binomial also forms a pattern. Have each member of your group find the fourth power of a binomial of their own choosing.

   a. Record and compare the products.

   Answers will vary.
   For example: $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

   b. Describe any patterns you notice.

   Answers will vary.

   For example, the fourth powers of both terms of the binomial are terms of the product, the product has 5 terms, or the coefficients of the second and fourth terms are multiples of 4 and the coefficient of the third term is a multiple of 6.

5. Use the pattern you described in problem 4 to find the product of $(2+3a)^4$.

   $16 + 96a + 216a^2 + 216a^3 + 81a^4$

6. What do you predict about the product of $(x+y)^5$? Explain.

   Answers will vary.

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LAB 7–A

Name _____

Date _____

POWER BINGO 1

Materials: Game sheet, notebook paper, beans or paper clips to serve as markers (optional), and a sack containing four each of tiles labeled $3x$, $3y$, x^2, and y^2

Power Bingo uses game sheets of 16 squares containing monomials that are the **products** of any three of the following monomials:

$3x$	$3x$	$3x$	$3x$	$3y$	$3y$	$3y$	$3y$
x^2	x^2	x^2	x^2	y^2	y^2	y^2	y^2

Three tiles are drawn from the sack, and the three monomials picked are announced to the class and displayed on the overhead. If a square on your game sheet contains the product of the three monomials announced, you may cover or mark that square. The first player to cover four squares in a row vertically, horizontally, or diagonally wins the game.

1. One possible monomial is $9x^3y$, the product of $3x$, $3y$, and x^2. How many possible products do you think there are? How did you arrive at your answer?

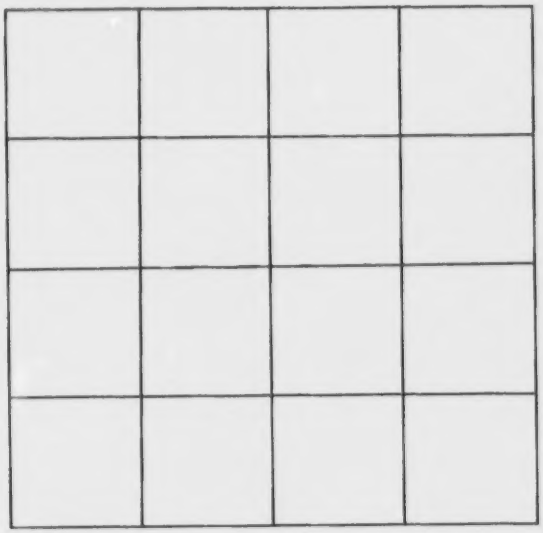
2. Find all possible monomials that are products of any three of the 16 monomials listed above. Write your products on notebook paper.

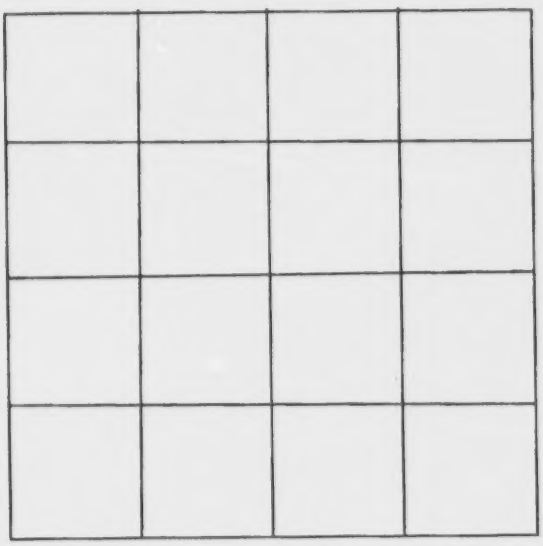
3. How does the number of monomials in part 2 compare with your answer in part 1?

4. You are almost ready to play Power Bingo. Before you can play, you need to fill your game sheet with different possible products of three monomials that might be drawn from the sack. When you are done, you are ready to play.

6. Looking back: Now that you have played one game of Power Bingo, what changes might you make in filling out your game sheet? Why?

POWER BINGO GAME SHEET





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LAB 7–B

POWERS OF BINOMIALS

Name _____

Group members _____

_____

_____

Date _____

1. Have each member of your group select one of the problems below and find the cube of the binomial. Share your results and write the products in descending powers of the variable.

   a. $(x + y)^3$

   b. $(p + 2)^3$

   c. $(3 + m)^3$

   d. $(c - d)^3$

2. The product obtained by cubing a binomial forms a pattern. Compare your results from problem 1 with your group members' results and describe any patterns you notice.

3. Use the pattern you described in problem 2 to find the cube of each binomial below:

   a. $(m - 1)^3$

   b. $(j + 2k)^3$

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4. The product obtained by taking the fourth power of a binomial also forms a pattern. Have each member of your group find the fourth power of a binomial of their own choosing.

   a. Record and compare the products.

   b. Describe any patterns you notice.

5. Use the pattern you described in problem 4 to find the product of $(2 + 3a)^4$.

6. What do you predict about the product of $(x + y)^5$? Explain.

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Chapter 8 Products and Factors

Teacher Notes

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Student Worksheet Masters

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LAB 8–A NOTES

TILE MANIA

Purpose: Using manipulatives

Tile Mania uses a game format to allow students to model quadratic products.

Materials: Three specially marked dice, set of Algebra Tiles, notebook paper, Tile Mania directions sheet, and Tile Mania student worksheet (one sheet per student; you may want to pass out at the end of the activity).

Directions: Students play in pairs. Be sure they understand the instructions on the Tile Mania activity sheet. A review of the area, length, and width of each algebra tile may be helpful.

Have students consider a sample roll of the dice. Can all the pieces be used to form a rectangle? Have students discuss and try in their groups. Have groups share the rectangles they formed and the multiplication/division fact teams corresponding to their rectangles. Go over the scoring for each rectangle shared.

Have the groups play at least two rounds of the Tile Mania game and record their outcomes.

During play, the issue of how to score rectangles that are squares may arise. The class will have to resolve how to award points for multiplication/division fact teams that have only two equations.

Have groups share some of their outcomes. What would you consider a good roll? What kind of roll would not be good? Assign the remainder of the Tile Mania student worksheet.

If time allows, you may want to make this a two-day activity.

For the first day have students record only the outcome of their three-dice roll and sketch the rectangular array they formed.

On the second day students can write the fact team and the score for each rectangle, discuss some of their outcomes, and finish the Tile Mania student worksheet.

TILE MANIA DIRECTIONS

Object: To form a square or rectangular array using the greatest number of tiles.

Materials: Three specially marked dice, pack of Algebra Tiles

Players: Two

Method of play: Player 1 throws all three dice, then selects the tiles matching the values thrown. Player 1 builds a rectangular or square array using as many of the tiles as possible. At least one x^2 tile must be used. Player 1 sketches the array and writes the multiplication/division fact team represented by the array.

Player 2 then builds a *different* array using the tiles from Player 1's dice throw, sketches the array, and writes the multiplication/division fact team represented by the array.

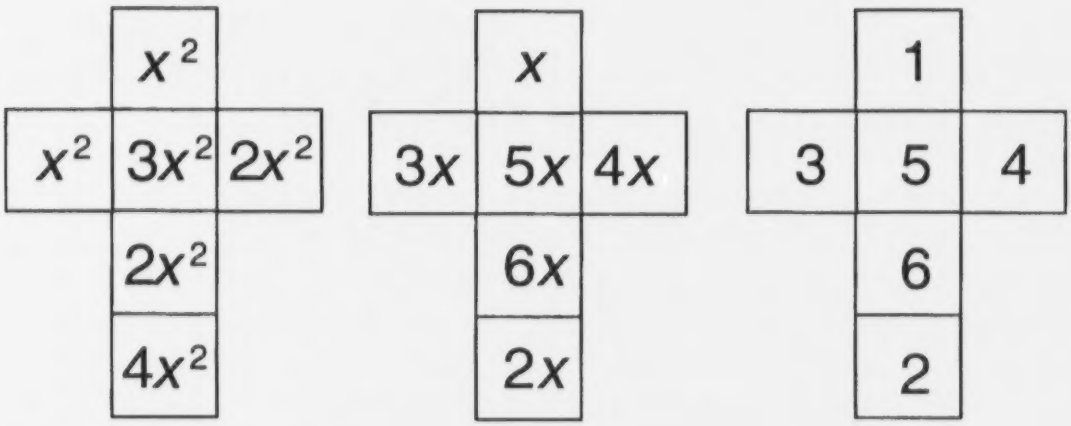
Play continues. This time Player 2 throws the dice. Players alternate throwing the dice to build the arrays.

Scoring: Earn 2 points for each correct multiplication and division sentence.
Earn 1 point for each tile used.
Deduct 1 point for each tile *not* used.
The first player to score 40 points wins.

Variation: The player with the highest score after a set number of rounds wins.

Construction of dice: Make the three dice as shown on the next page.

LAB 8–A NOTES



1. a. Suppose you rolled $2x^2$, $5x$, and 4. Sketch the rectangular or square array that uses as many tiles as possible.

b. Write the multiplication/division fact team for your array.

$(x + 1)(2x + 3) = 2x^2 + 5x + 3$
$(2x + 3)(x + 1) = 2x^2 + 5x + 3$
$(2x^2 + 5x + 3) \div (2x + 3) = x + 1$
$(2x^2 + 5x + 3) \div (x + 1) = 2x + 3$

c. What will be your score for the round?

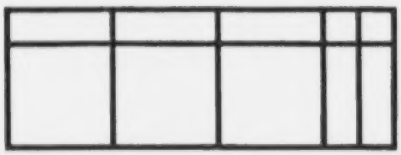
17 points (8 points for fact team equations, 10 points for tiles used, –1 point for tile not used)

2. Ryan needs at least 15 points to take the lead from Ian. Is this possible? Justify your answer.

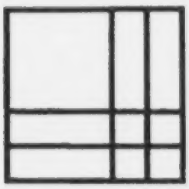
Answers will vary. Diagrams/fact teams from problem 1 may be referenced.

3. Teiti said that she was able to use all of her tiles. Find two possible rolls Teiti could have gotten. Sketch diagrams for arrays she could form and write the corresponding fact teams.

Answers will vary. For example, Teiti could have rolled $3x^2$, $5x$, and 2 or x^2, $4x$, and 4.



$(3x + 2)(x + 1) = 3x^2 + 5x + 2$
$(x + 1)(3x + 2) = 3x^2 + 5x + 2$
$(3x^2 + 5x + 2) \div (3x + 2) = x + 1$
$(3x^2 + 5x + 2) \div (x + 1) = 3x + 2$



$(x + 2)^2 = x^2 + 4x + 4$
$(x^2 + 4x + 4) \div (x + 2) = x + 2$

4. Write down any observations you made while playing the game. For example, what kinds of rolls were "good" and what kinds didn't you want to get?

Answers will vary.

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LAB 8–B NOTES

POWER BINGO 2

Purpose: Extending concept development

Power Bingo 2 uses a game format that lets students practice finding the product of two binomials.

Materials: Power Bingo game sheet, Power Bingo 2 sheet (one per student), Power Bingo 2 worksheet (optional)

Directions: Students should find the products using the binomials as homework before playing Power Bingo 2. If students come to class unprepared, you may want them to complete the Power Bingo 2 worksheet while the class plays the game.

Make four copies of each binomial factor on transparency pieces and put them in a sack. From the sack, randomly select, announce, and display the transparency pieces on the overhead. Keep a record of the products to verify winners' bingos.

Variations may include

- filling the second game square allowing duplicate products entered in the cells. However, each product can be covered only one at a time.
- pulling two binomials at a time but not replacing them in the sack until two binomials can no longer be pulled.
- winning with four corners or a 2-by-2 square anywhere on the game square.

LAB 8–A

TILE MANIA

Name _____

Partner _____

Date _____

1. a. Suppose you rolled $2x^2$, $5x$, and 4. Sketch the rectangular or square array that uses as many tiles as possible.

   b. Write the multiplication/ division fact team for your array.

   c. What will be your score for the round?

2. Ryan needs at least 15 points to take the lead from Ian. Is this possible? Justify your answer.

3. Teiti said that she was able to use all of her tiles. Find two possible rolls Teiti could have gotten. Sketch diagrams for arrays she could form and write the corresponding fact teams.

4. Write down any observations you made while playing the game. For example, what kinds of rolls were "good" and what kinds didn't you want to get?

LAB 8–B

Name _____

Date _____

POWER BINGO 2

Materials: Power Bingo game sheet

Tomorrow we will play Power Bingo 2. This time you will fill out your Power Bingo game square with 16 polynomials that are the products of two binomials. These are binomials you can use to create your polynomials:

$x + 1$	$x + 4$	$x - 1$	$x - 4$
$x + 2$	$x + 5$	$x - 2$	$x - 5$
$x + 3$	$x + 6$	$x - 3$	$x - 6$

Here are 3 of the 16 polynomials Mike used on his Power Bingo card:

$x^2 - 4$ $\qquad$ $x^2 + 6x + 9$ $\qquad$ $x^2 - 3x - 10$

Mike got these polynomials by taking the following products of two binomials:

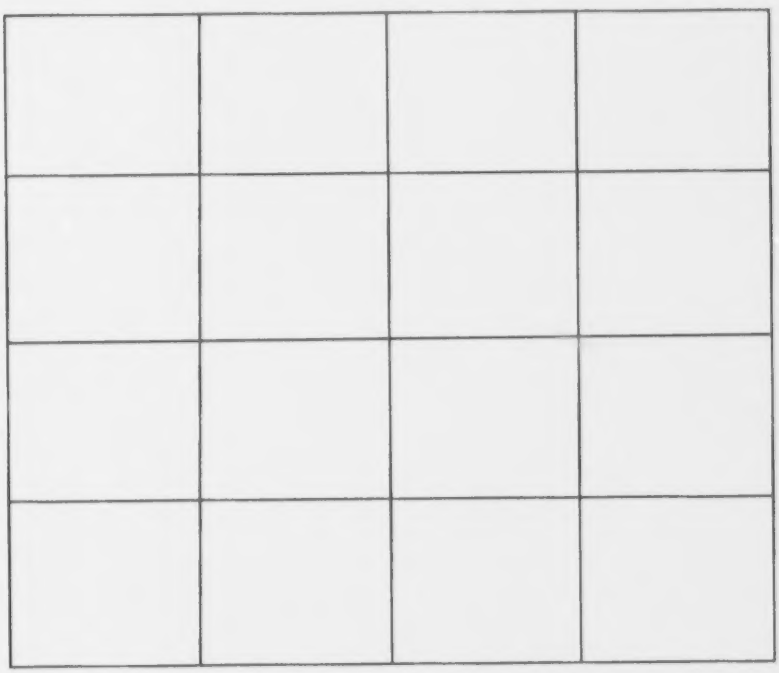
$(x + 2)(x - 2)$ $\qquad$ $(x + 3)(x + 3)$ $\qquad$ $(x - 5)(x + 2)$

As you can see, a binomial may be used more than once.

Your homework tonight will be to come up with 16 polynomials, each polynomial being the product of two binomials listed above. Be sure to double-check your multiplication.

Then place the 16 polynomials on your Power Bingo card so that you will be ready to play the game tomorrow.

POWER BINGO GAME SHEET



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LAB 8–B

Name _____

Power Bingo 2 Worksheet

Find the following products:

1. $(x + 1)(x - 2)$
2. $(x + 1)(x + 4)$
3. $(x - 1)(x - 3)$
4. $(x - 1)^2$
5. $(x + 2)(x - 2)$
6. $(x + 2)(x + 3)$
7. $(x + 2)^2$
8. $(x - 2)(x - 4)$
9. $(x - 2)(x + 3)$
10. $(x + 3)(x - 4)$
11. $(x + 3)(x + 4)$
12. $(x - 3)(x - 2)$
13. $(x - 3)^2$
14. $(x + 4)(x + 2)$
15. $(x + 4)(x - 3)$
16. $(x - 4)(x - 1)$
17. $(x - 4)(x + 1)$
18. $(x + 4)(x - 4)$

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Chapter 9 Quadratic Equations

Teacher Notes

Lab 9–A	Sew, Sew	9–1
Lab 9–B	Polygons and Diagonals	9–3

Student Worksheet Masters

Lab 9–A	Sew, Sew	9–5
Lab 9–B	Polygons and Diagonals	9–7

LAB 9–A NOTES

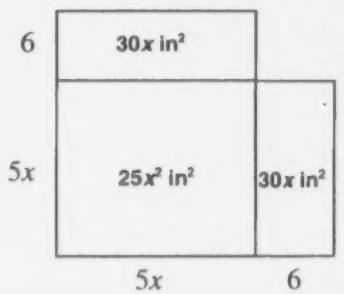
SEW, SEW

Purpose:	Extending concept development Sew, Sew allows students to explore completing the square using the context of making a quilt.
Materials:	Sew, Sew sheet (one per student), pack of Algebra Tiles
Directions:	Place students in groups of two, three, or four and distribute materials. Tell students they are to answer each question individually but they can solve the problem within their group. Allow students 25–30 minutes to complete. Before the period ends, have students share their solutions.

Taylor had several pieces of fabric he was sewing into a square blanket. The diagram shows three pieces that he has laid out with measurements in inches.

1. What is the area of each piece he has laid out?

   See diagram



2. What is the area of the piece he needs to complete his square blanket?

   The piece he needs must be 36 in².

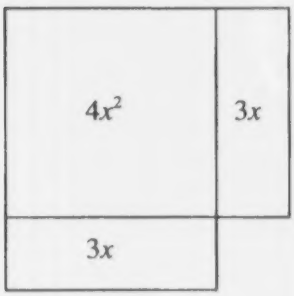
3. The total area of the completed blanket is 4,356 square inches. What is the value of x?

   The value of x is 12 inches.

4. The total area of the three pieces Taylor started out with was 2,565 square inches. What is the value of x?

The value of x is $2\frac{1}{5}$ inches.

The diagram below depicts part of a square blanket that Jessica is designing. The area of each panel is indicated in square feet.



5. What is the area of the small piece that would complete Jessica's square blanket? Explain how you got your answer.

The area of the small piece is $2\frac{1}{4}$ ft^2. Methods will vary.

6. Jessica wants her completed square blanket to have a total area of 49 square feet. What should the value of x be?

The value of x is $2\frac{3}{4}$ ft.

7. Create a "square blanket" problem similar to Taylor's and Jessica's. Include your solution.

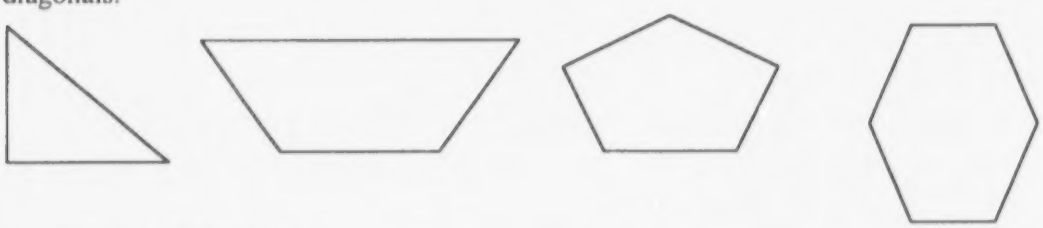
Answers will vary. Students should include a correct solution.

LAB 9–B NOTES

POLYGONS AND DIAGONALS

Purpose:	Extending concept development
	Polygons and Diagonals allows students to examine data which fit a quadratic pattern.
Materials:	Polygons and Diagonals sheet (one per student), graph paper, ruler or straightedge
Directions:	Students work in pairs but each person should answer the problems individually.
	Allow about 10 minutes at the end of the period for students to discuss patterns they noticed.

Jake discovered a relationship between the number of sides in a polygon and the number of its diagonals.



1. Complete Jake's table:

Number of Sides (x)	Number of Diagonals (y)
3	0
4	2
5	5
6	9
7	14
8	20
9	27

2. Describe patterns you notice in the table.

Answers may vary. For example, the number of diagonals increases by consecutive numbers.

LAB 9–B NOTES

3. Write an equation whose graph would include the points given by the data in your table. Graph your equation.

Equations should be equivalent to $y = \frac{x(x-3)}{2}$.

4. A polygon has 17 sides. How many diagonals will it have? Describe your reasoning.

A 17-sided polygon will have 119 diagonals. Students' reasoning may include substituting 17 for x in the equation for problem 3 and solving for y.

5. A polygon has 209 diagonals. How many sides does it have? Describe your reasoning.

A polygon with 209 diagonals must have 22 sides. Students may have used the equation from problem 3 or continued filling the table of values.

LAB 9–A

SEW, SEW

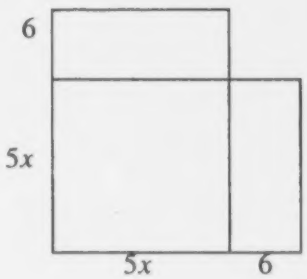
Name _____

Date _____

Materials: Pack of Algebra Tiles

Taylor had several pieces of fabric he was sewing into a square blanket. The diagram shows three pieces that he has laid out with measurements in inches.

1. What is the area of each piece he has laid out?

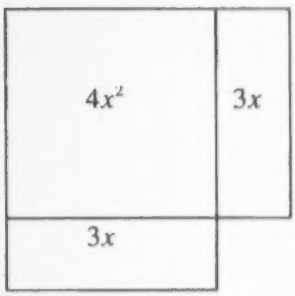


2. What is the area of the piece he needs to complete his square blanket?

3. The total area of the completed blanket is 4,356 square inches. What is the value of x?

4. The total area of the three pieces Taylor started out with was 2,565 square inches. What is the value of x?

The diagram below depicts part of a square blanket that Jessica is designing. The area of each panel is indicated in square feet.



5. What is the area of the small piece that would complete Jessica's square blanket? Explain how you got your answer.

6. Jessica wants her completed square blanket to have a total area of 49 square feet. What should the value of x be?

7. Create a "square blanket" problem similar to Taylor's and Jessica's. Include your solution.

LAB 9–B

POLYGONS AND DIAGONALS

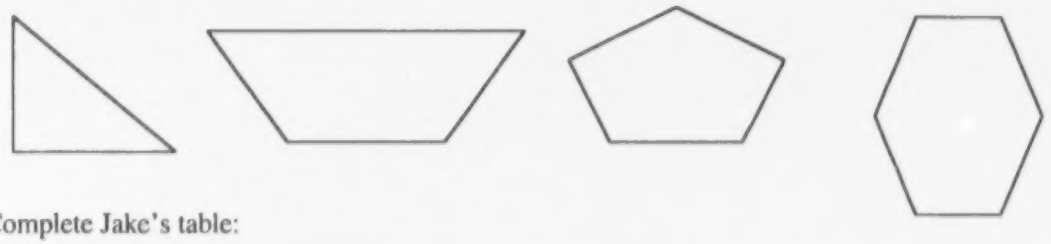
Name _____

Partner _____

Date _____

Materials: Graph paper, ruler or straightedge

Jake discovered a relationship between the number of sides in a polygon and the number of its diagonals.



1. Complete Jake's table:

Number of Sides (x)	Number of Diagonals (y)
3	
4	
5	
6	
7	
8	
9	

2. Describe patterns you notice in the table.

3. Write an equation whose graph would include the points given by the data in your table. Graph your equation.

4. A polygon has 17 sides. How many diagonals will it have? Describe your reasoning.

5. A polygon has 209 diagonals. How many sides does it have? Describe your reasoning.

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Chapter 10 Rational Expressions and Equations

Teacher Notes

Lab 10–A	Design-a-Box Contest	10–1
Lab 10–B	Tube Scenes	10–4
Lab 10–C	The Party Bill Problem	10–5

Student Worksheet Masters

Lab 10–A	Design-a-Box Contest	10–7
Lab 10–B	Tube Scenes	10–9
Lab 10–C	The Party Bill Problem	10–11

LAB 10–A NOTES

DESIGN-A-BOX CONTEST

Purpose: Problem solving

Design-a-Box is in a contest format where students design a rectangular box and may work with a third-degree equation.

Materials: Design-a-Box Contest (one sheet per student), sheets of $8\frac{1}{2}$" by 11" paper, scissors, rulers with inch markings, tape, graph paper, graphing calculators

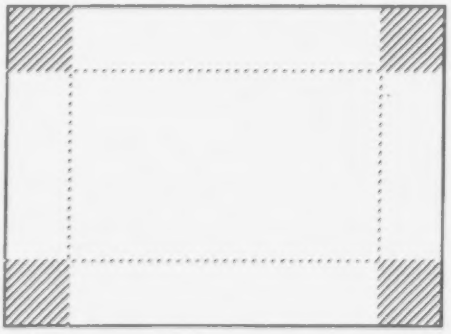
Directions: Have students work in pairs. Every pair should have a sheet of paper, scissors, a ruler, and enough tape to secure their box. Go over the rules of the contest and tell students that they will have only a five-minute time limit (although you may give them more time in reality). Announce when there are two minutes and one minute left. If pairs are slow to start, you may have to remind the class that only finished boxes will be allowed to enter the contest.

When time is called, have students write their names in their boxes and register their entry with you. (Write a number on one side of the box so that the number will be visible.) Now the second contest can begin. Have students hold up the boxes so that everyone can see the box's number. Have students fill out their Guess-the-Winner entry blank. Collect all entries.

Have each pair find the height (in inches) and volume (in cubic inches) of their box. The data should be recorded in a chart at the front of the room.

Sometimes data will have the same height, but different volumes. Do not point this out. Wait and see what students say. Don't worry if no one notices anything. Someone usually will point out the discrepancy after they graph the data and work on Part II of the Lab.

The Algebra Box Company (ABC) is sponsoring a design-a-box contest. The winner is the contestant who makes a box from an $8\frac{1}{2}$ by 11-inch sheet of paper with the greatest volume. The box must be an open box made by cutting an identical square from each corner and folding the edges to form the box.



LAB 10–A NOTES

Part I

1. Work with a partner to make a box to enter the contest. You will have only 5 minutes to make your box.

2. Place your box in the display area. Compare your box with other entries and predict which box has the greatest volume. Record your prediction on the entry blank provided. (There is a separate Guess-the-Winner contest. Deadline to enter the Guess-the-Winner contest will be announced by your teacher.)

   Answers will vary.

3. Measure your box's height, length, and width in inches. Determine the volume of your box in cubic inches. Record your box's height and volume on the chart at the front of the room.

   Answers will vary.

4. Graph the data from the chart.

   Answers will vary. Graph should match data recorded in chart.

Part II

1. Describe how you and your partner decided on what your box would look like.

   Answers will vary. Watch for logical solution processes.

2. Refer to your graph. How did you decide on the units for your horizontal and vertical axes? Describe what your graph looks like. What conclusions can you make?

   Answers will vary. Description and conclusions should match graph.

3. Do you think any of the boxes constructed had the greatest possible volume? Support your answer.

   Answers will vary. Responses must be supported by a valid mathematical argument.

4. Renee said that a graphing calculator can be used to help find the box with the greatest possible volume. What do you think she meant by this?

Answers will vary.

LAB 10–B NOTES

TUBE SCENES

Purpose: Extending concept development

Tube Scenes allows students to gather data, look for a pattern, and make a prediction based on that pattern.

Materials: Tube Scenes sheet (one per student), tape measure, yard or meter stick, masking tape, toilet tissue cardboard tube

Directions: Place students in groups of four. Students work in groups, but each person should answer the problems individually. Go over the procedures with the class so students understand which distances they will measure.

Allow 5–10 minutes at the end of the period for students to discuss their findings.

Data will vary.

Range of View	Distance from Meter Stick to Eye

6. Predict the distance you would have to be from the wall to see a range of only four inches. Explain how you arrived at your answer.

Answers will vary.

7. Check your prediction by actually finding the best location.

Answers will vary. Students should describe the location they determine as the best.
To extend the lab, ask students to predict where they would need to stand to see the entire meter stick. Have them check predictions and describe how they made their predictions.

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LAB 10–C NOTES

THE PARTY BILL PROBLEM

Purpose:	Problem solving
	Students have an opportunity to use rational equations as a method to solve the problem.
Materials:	The Party Bill Problem sheet (one per student)
Directions:	Students work in pairs, but each person should answer the problems individually.
	Allow 5–10 minutes at the end of the period for students to discuss their solutions.

1. The attendees of the Hardy party were going to pay equal shares of their restaurant bill of \$120. But when they were leaving, they discovered that two members of the party had slipped away without paying (See if they are invited again!). This meant that each of the rest had to pay \$5 more than they expected. How many people were originally in the party?

   Eight people were originally in the party.
   Methods will vary. For example, an algebraic method where x represents the original number of people in the party and the equation $\frac{120}{x} = \frac{120}{x-2} + 5$ might be used.

2. Use another method (excluding guess-and-test) to solve the problem.

   Answers will vary. For example, if x represents the number of people who actually paid, the equation used might be $\frac{120}{x} + 5 = \frac{120}{x+2}$.

3. How are the two methods similar? How are they different?

   Answers will vary.

LAB 10–A

DESIGN-A-BOX CONTEST

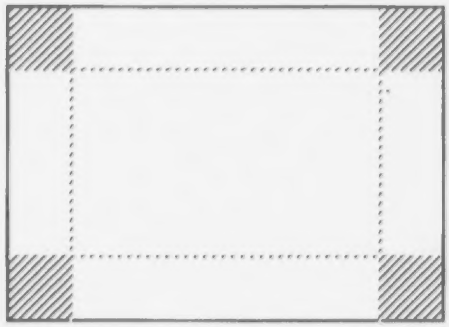
Name _____

Partner _____

Date _____

Materials: $8\frac{1}{2}$" by 11" paper, scissors, ruler with inch markings, tape, graph paper, graphing calculator

The Algebra Box Company (ABC) is sponsoring a design-a-box contest. The winner is the contestant who makes a box from an $8\frac{1}{2}$" by 11-inch sheet of paper with the greatest volume. The box must be an open box made by cutting an identical square from each corner and folding the edges to form the box.



Part I

1. Work with a partner to make a box to enter the contest. You will have only 5 minutes to make your box.

2. Place your box in the display area. Compare your box with other entries and predict which box has the greatest volume. Record your prediction on the entry blank provided. (There is a separate Guess-the-Winner contest. Deadline to enter the Guess-the-Winner contest will be announced by your teacher.)

3. Measure your box's height, length, and width in inches. Determine the volume of your box in cubic inches. Record your box's height and volume on the chart at the front of the room.

4. Graph the data from the chart.

Part II

1. Describe how you and your partner decided on what your box would look like.

2. Refer to your graph. How did you decide on the units for your horizontal and vertical axes? Describe what your graph looks like. What conclusions can you make?

3. Do you think any of the boxes constructed had the greatest possible volume? Support your answer.

4. Renee said that a graphing calculator can be used to help find the box with the greatest possible volume. What do you think she meant by this?

Design-a-Box
Guess-the-Winner Contest

Name_____

I predict that Box No. _____ has the greatest volume.

I estimate that the volume is _____ cubic inches.

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LAB 10–B

TUBE SCENES

Name _____

Group members _____

_____

_____

Date _____

Materials: Tape measure, yard or meter stick, masking tape, toilet tissue cardboard tube

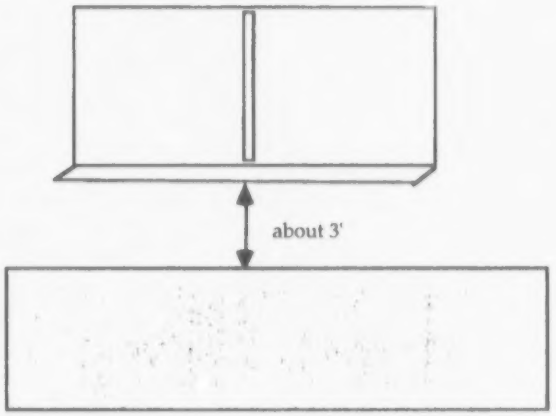
1. Use masking tape to secure a measuring stick vertically on the wall or chalkboard. Be sure it is about three feet off the floor.

2. Have team member A raise the tubing to his/her eye, keeping it parallel to the floor. Stand about three feet from the wall. Adjust your distance to the wall until you can see a portion of the measuring stick.

3. Report the range you can see. Do not move from your position. Have two other members of the group use a tape measure to measure the distance from the meter stick to your eye.

4. Record both measurements in the table below.

5. Have member A move closer to the wall until he/she can see only half the range. Measure the distance from the meter stick to your eye. Record the measurements in the table below.



Range of View	Distance from Meter Stick to Eye

6. Predict the distance you would have to be from the wall to see a range of only four inches. Explain how you arrived at your answer.

7. Check your prediction by actually finding the best location.

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LAB 10–C

THE PARTY BILL PROBLEM

Name _____

Partner _____

Date _____

1. The attendees of the Hardy party were going to pay equal shares of their restaurant bill of $120. But when they were leaving, they discovered that two members of the party had slipped away without paying (See if they are invited again!). This meant that each of the rest had to pay $5 more than they expected. How many people were originally in the party?

2. Use another method (excluding guess-and-test) to solve the problem.

3. How are the two methods similar? How are they different?

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   Ill., 2nd ed., 2001, hard, ISBN 1-58351-008-7, $40.00

2. **Algebra I: A Process Approach Teacher's Guide**
   Annette Matsumoto, Barbara J. Dougherty, Li Ann Wada, Sidney L. Rachlin, Fay Zenigami
   Ill., 2nd ed., 2001, loose-leaf with binder, ISBN 1-58351-011-7, $99.00

3. **Algebra I: A Process Approach Teacher Resource Set**
   Barbara J. Dougherty, Annette Matsumoto, Li Ann Wada, Fay Zenigami, Sidney L. Rachlin, Jenny Simmons
   Ill., 2nd ed., 2001, loose-leaf with binder, ISBN 1-58351-013-3, $95.00

4. **Explorations in Algebra**
   Barbara J. Dougherty, Annette Matsumoto, Fay Zenigami
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